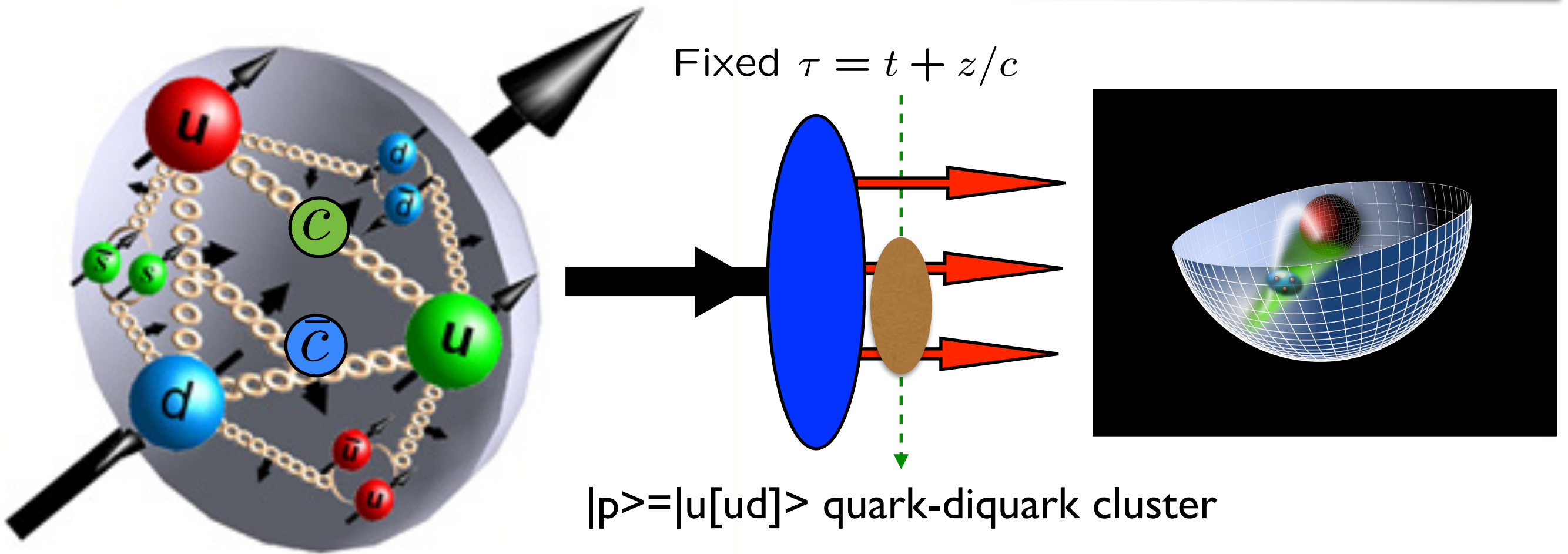


New Perspectives for Hadron Spectroscopy and Dynamics and the Running QCD Coupling from Color-Confining Holographic Light-Front QCD



with Guy de Tèramond, Hans Günter Dosch, Cédric Lorcè, Alexandre Deur, and Joshua Erlich

A.N. Mitra
Memorial Symposium
April 15, 2025

Stan Brodsky
SLAC NATIONAL
ACCELERATOR
LABORATORY



A.N. Mitra

**QCD at the amplitude level
Factorization Theorems,
Counting Rules,
Light-Front Theory
Nuclear Amplitudes**

Spin dynamics of qqq wave function on light front in high momentum limit of QCD: Role of qqq force

A.N. Mitra

Annals Phys. 323 (2008) 845-865

Meson-Baryon Couplings in a Quark Model

A.N. Mitra
Delhi U.

,
Marc Ross
Michigan U.

Published in:

Phys.Rev. 158 (1967) 1630-1638

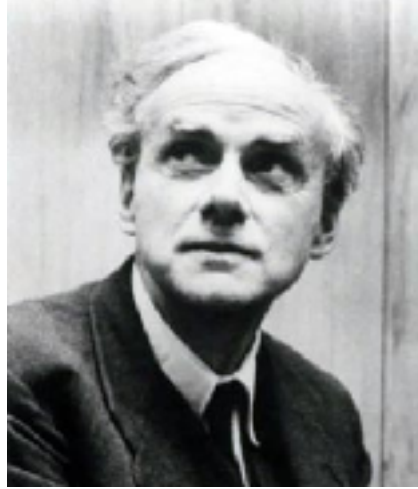
Relativistic Form-Factors for Clusters with Nonrelativistic Wave Functions

Asoke N. Mitra
SLAC

,
Indra Kumari
Delhi U.

Phys.Rev.D 15 (1977) 261

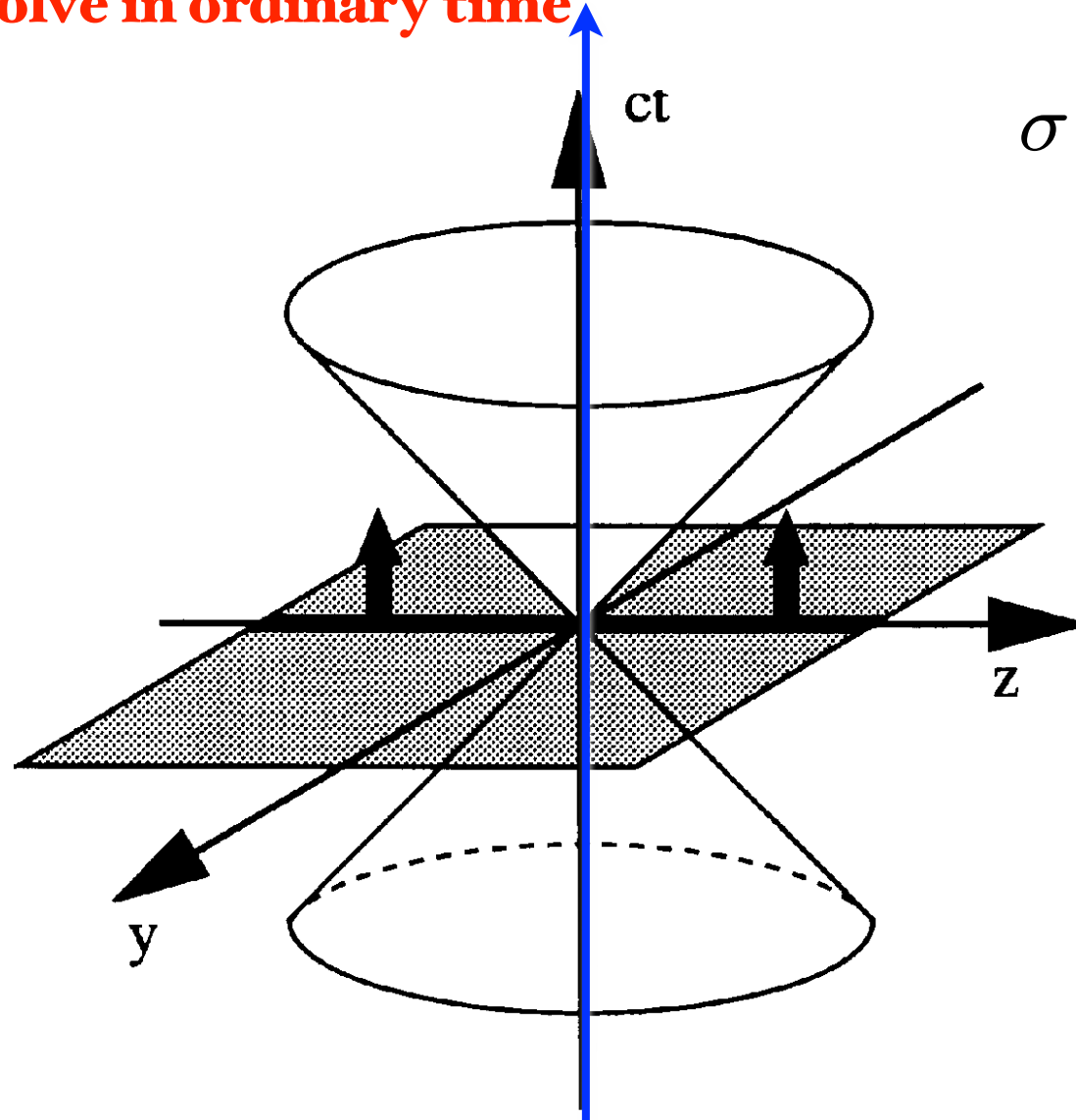
Light-Front Quantization



*P.A.M Dirac, Rev. Mod. Phys. 21,
392 (1949)*

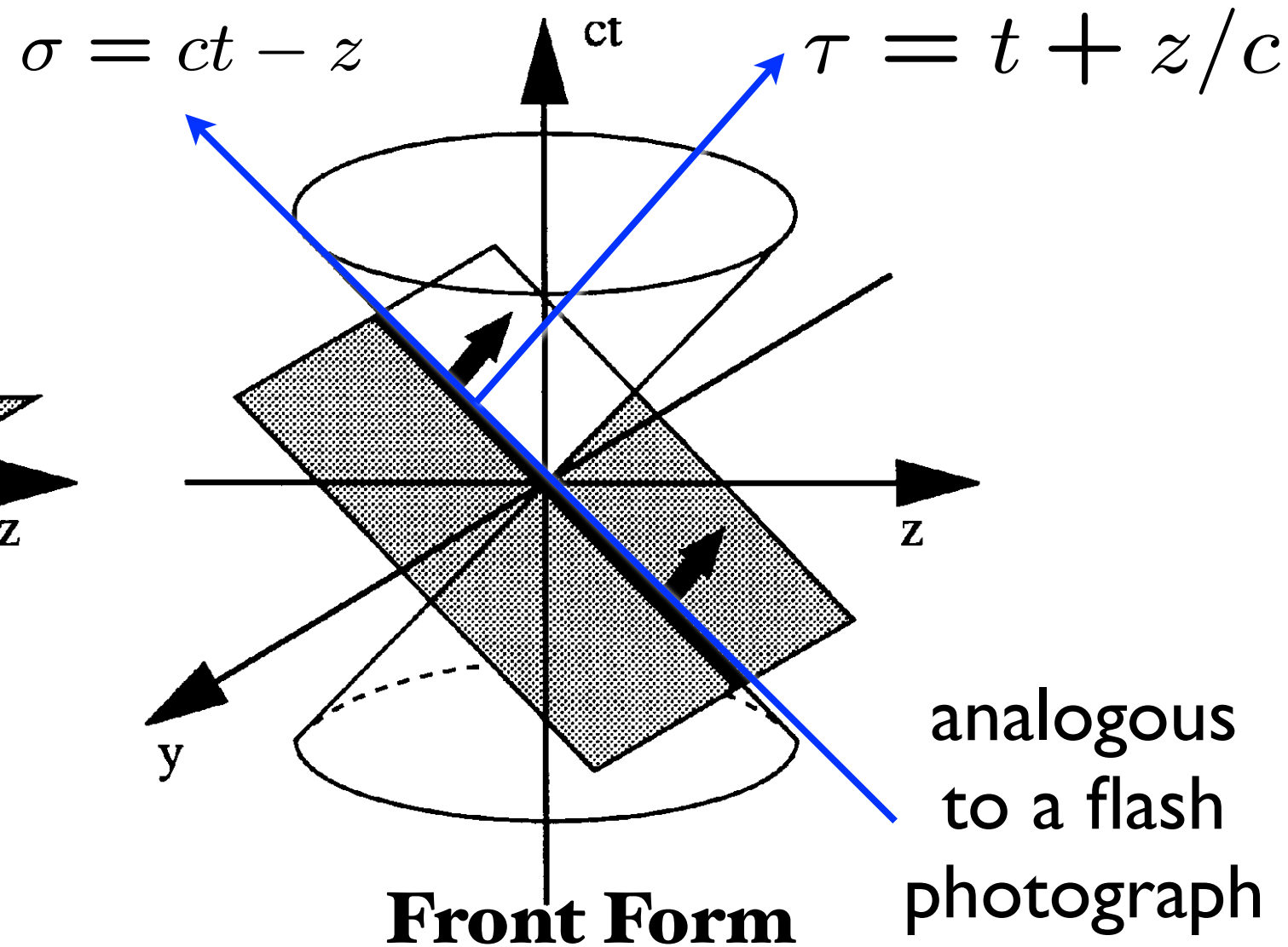
*Dirac's Amazing Idea:
The "Front Form"*

Evolve in ordinary time



Instant Form

Evolve in light-front time!



Front Form

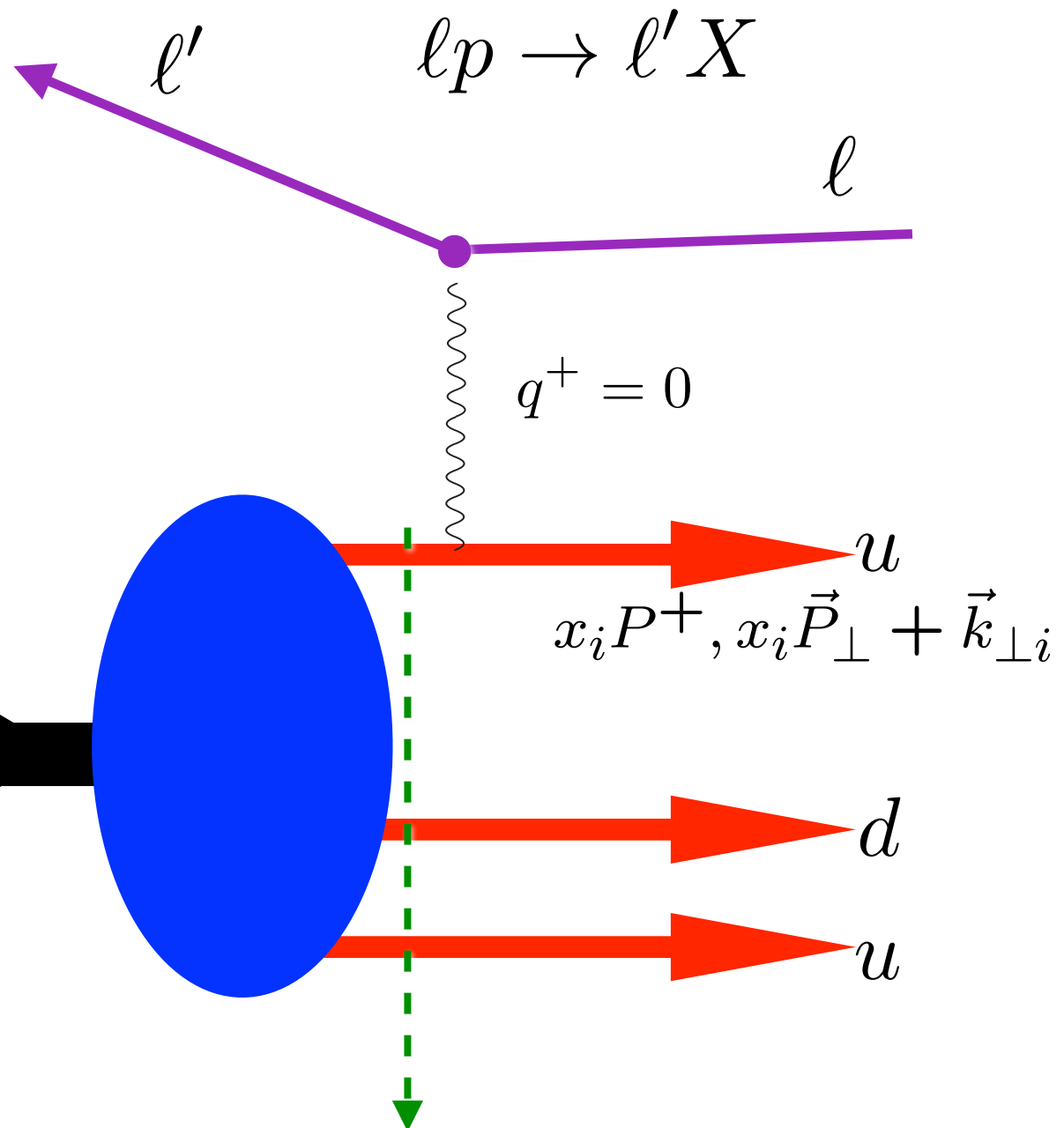
analogous
to a flash
photograph

Causal, Boost Invariant!

Comparing light-front quantization with instant-time quantization
Philip D. Mannheim(Connecticut U.),
Peter Lowdon(Ecole Polytechnique, CPHT),
Stanley J. Brodsky(SLAC)

• e-Print: [2005.00109](https://arxiv.org/abs/2005.00109) [hep-ph]

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



Dirac's Front Form

Measurements of hadron LF wavefunction are at fixed LF time

Fixed $\tau = t + z/c$

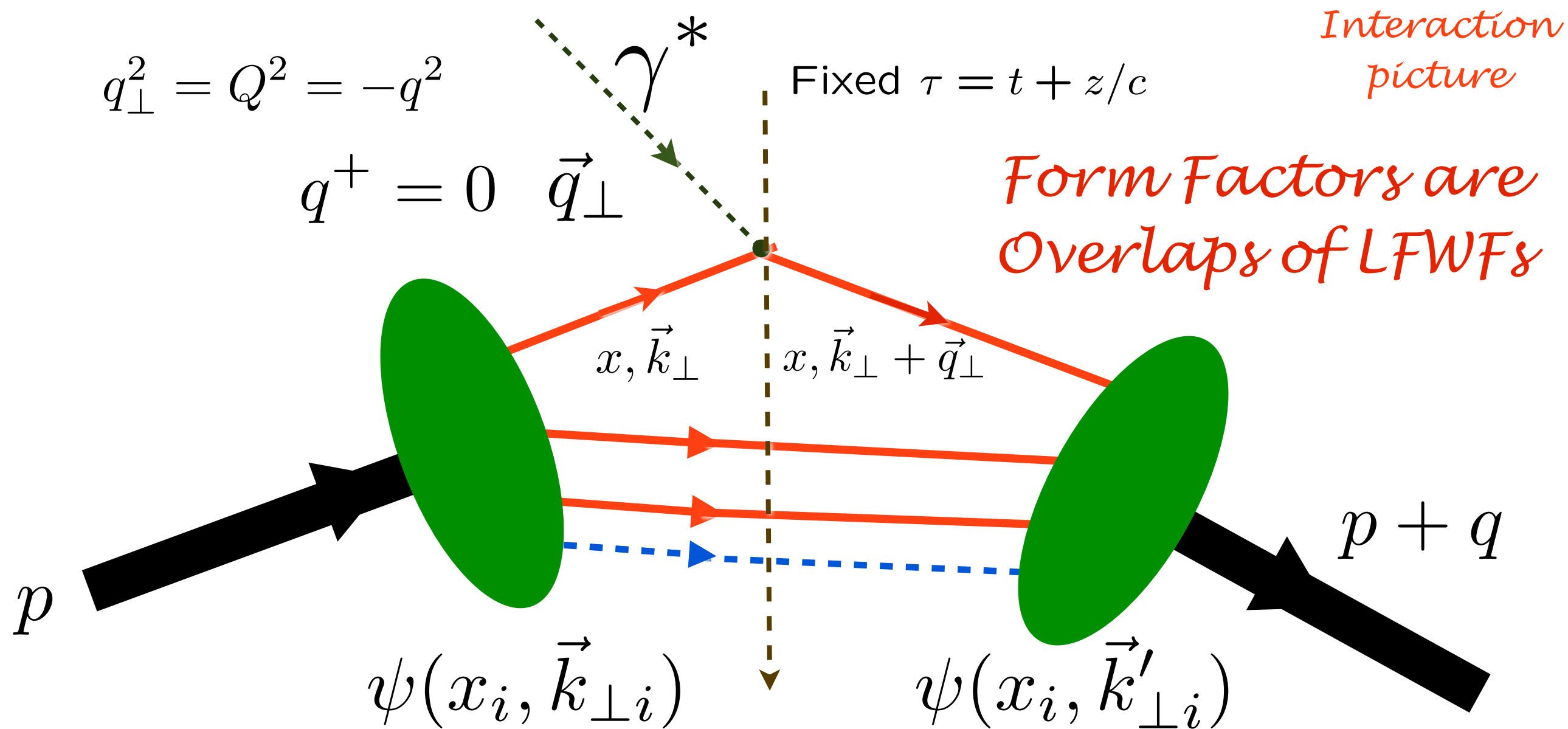
Like a flash photograph

$$x_{bj} = x = \frac{k^+}{P^+}$$

Invariant under boosts! Independent of P^μ

$$\langle p+q | j^+(0) | p \rangle = 2p^+ F(q^2)$$

Front Form

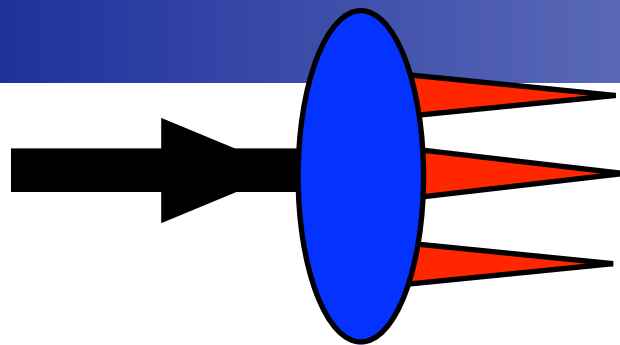


Drell & Yan, West
Exact LF formula!

struck $\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i) \vec{q}_{\perp}$
spectators $\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i \vec{q}_{\perp}$

Drell, sjb

Transverse size $\propto \frac{1}{Q}$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Light-Front Wavefunctions
underly hadronic observables

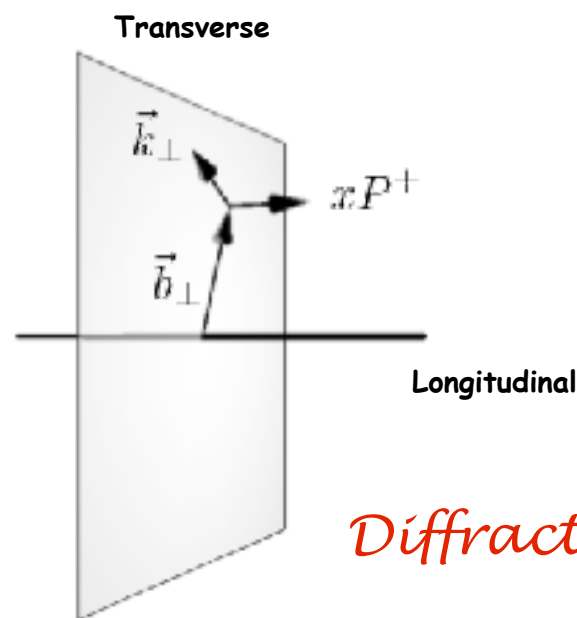
*Lorce,
Pasquini*

Momentum space $\vec{k}_{\perp} \leftrightarrow \vec{z}_{\perp}$ Position space
 $\vec{\Delta}_{\perp} \leftrightarrow \vec{b}_{\perp}$

Transverse density in
momentum space

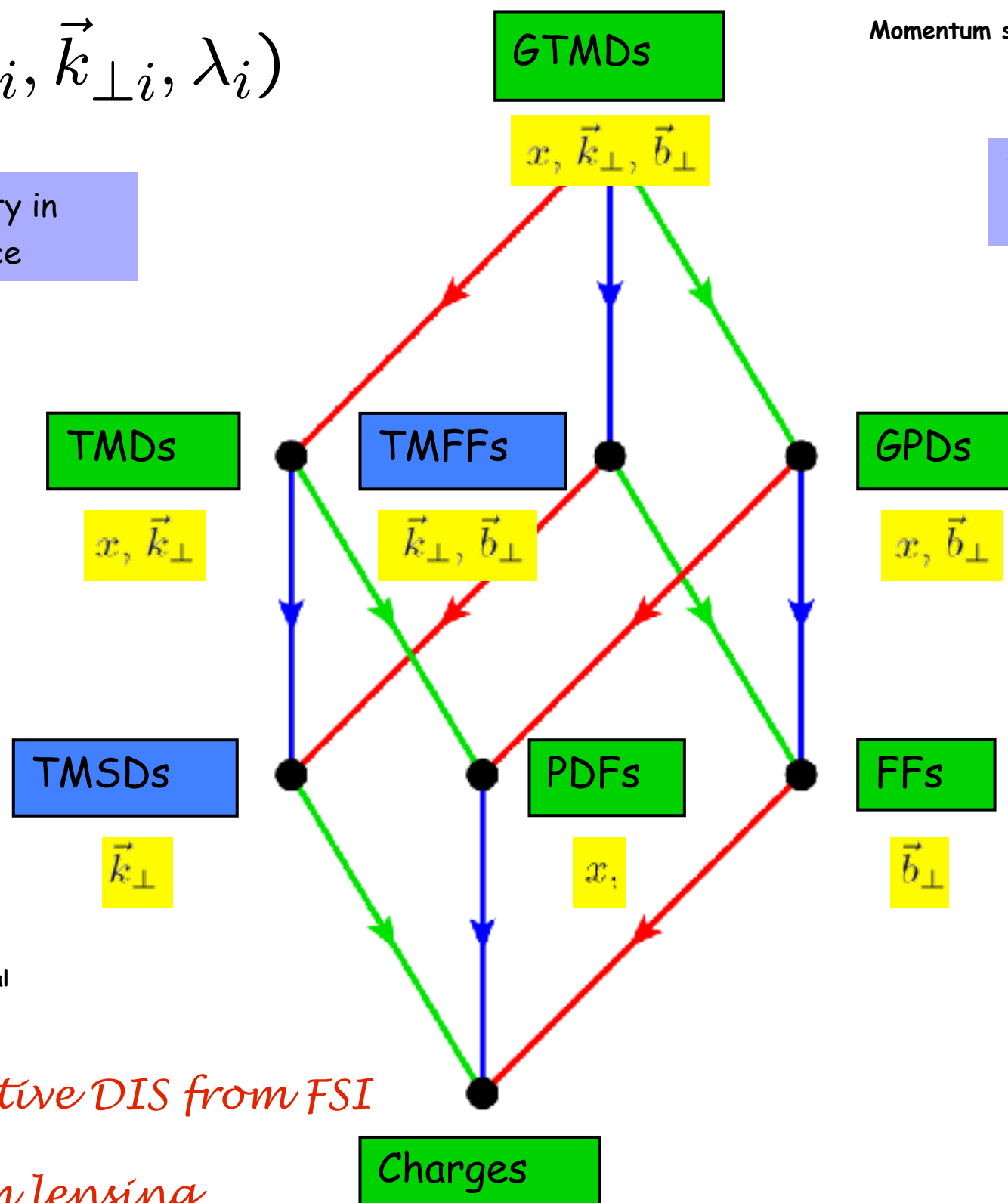
Transverse density in position
space

Weak transition
form factors



Diffractive DIS from FSI

Sivers, T-odd from lensing



*DGLAP, ERBL Evolution
Factorization Theorems*

\rightarrow $\int d^2 b_{\perp}$
 \rightarrow $\int dx$
 \rightarrow $\int d^2 k_{\perp}$

Exclusive processes in perturbative quantum chromodynamics

G. Peter Lepage

Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853

Stanley J. Brodsky

Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

(Received 27 May 1980)



We present a systematic analysis in perturbative quantum chromodynamics (QCD) of large-momentum-transfer exclusive processes. Predictions are given for the scaling behavior, angular dependence, helicity structure, and normalization of elastic and inelastic form factors and large-angle exclusive scattering amplitudes for hadrons and photons. We prove that these reactions are dominated by quark and gluon subprocesses at short distances, and thus that the dimensional-counting rules for the power-law falloff of these amplitudes with momentum transfer are rigorous predictions of QCD, modulo calculable logarithmic corrections from the behavior of the hadronic wave functions at short distances. These anomalous-dimension corrections are determined by evolution equations for process-independent meson and baryon “distribution amplitudes” $\phi(x_i, Q)$ which control the valence-quark distributions in high-momentum-transfer exclusive reactions. The analysis can be carried out systematically in powers of $\alpha_s(Q^2)$, the QCD running coupling constant. Although the calculations are most conveniently carried out using light-cone perturbation theory and the light-cone gauge, we also present a gauge-independent analysis and relate the distribution amplitude to a gauge-invariant Bethe-Salpeter amplitude.

Rigorous QCD analysis of exclusive reactions
Hadron Distribution amplitudes
ERBL Evolution

Also: Efremov and Radyshkin

Fixed $\tau = t + z/c$

Light-Front QCD

Physical gauge: $A^+ = 0$

Exact frame-independent formulation of
nonperturbative QCD!

$$L^{QCD} \rightarrow H_{LF}^{QCD}$$

$$H_{LF}^{QCD} = \sum_i \left[\frac{m^2 + k_{\perp}^2}{x} \right]_i + H_{LF}^{int}$$

H_{LF}^{int} : Matrix in Fock Space

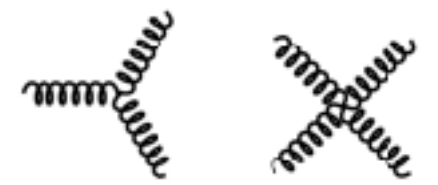
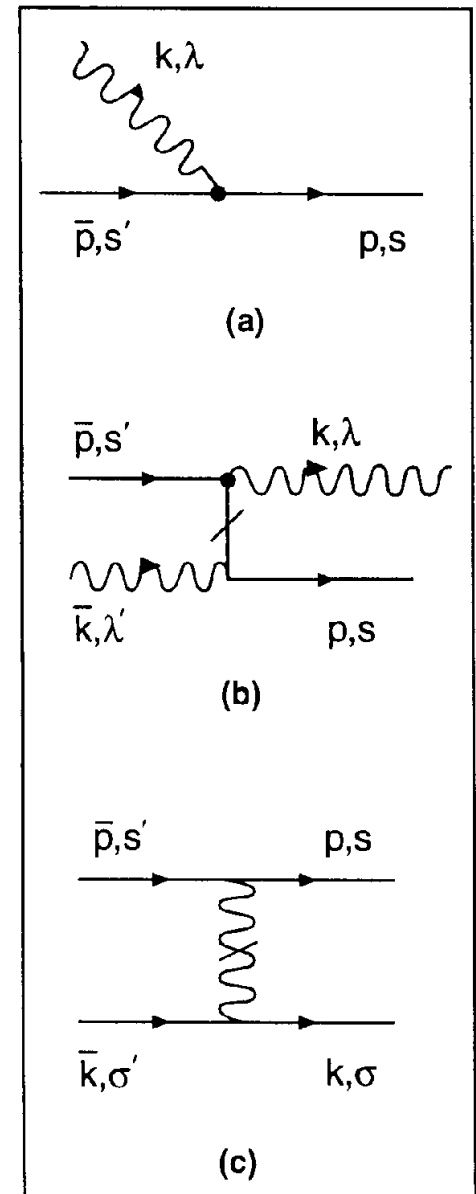
$$H_{LF}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle$$

$$|p, J_z\rangle = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i\rangle$$

Eigenvalues and Eigensolutions give Hadronic
Spectrum and Light-Front wavefunctions

LFWFs: Off-shell in P- and invariant mass

Solve nPQCD by matrix diagonalization: Hornbostel, Pauli, sjb



H_{LF}^{int}

Scaling: manifestation of asymptotically free hadronic interactions

From dimensional arguments at high energies in binary reactions:

CONSTITUENT COUNTING RULE

Brodsky and Farrar, Phys. Rev. Lett. 31 (1973) 1153
Matveev et al., Lett. Nuovo Cimento, 7 (1973) 719

Counting Rules:

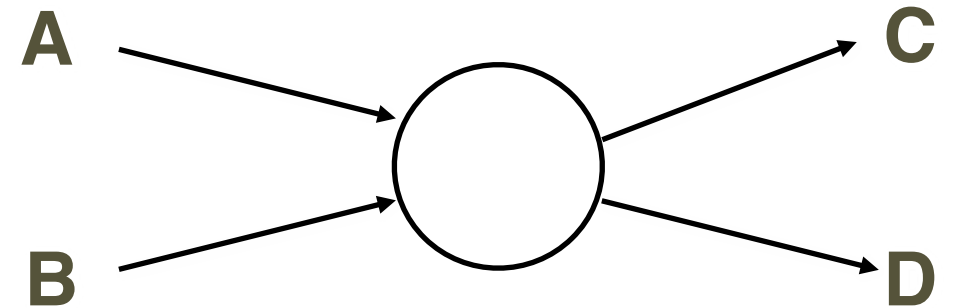
$$q(x) \sim (1-x)^{2n_{spect}-1} \text{ for } x \rightarrow 1$$

$$F(Q^2) \sim \left(\frac{1}{Q^2}\right)^{(n-1)}$$

$$\frac{d\sigma}{dt}(AB \rightarrow CD) \sim \frac{F(t/s)}{s^{(n_{participants}-2)}}$$

$$n_{participants} = n_A + n_B + n_C + n_D$$

$$\frac{d\sigma}{d^3p/E}(AB \rightarrow CX) \sim F(\hat{t}/\hat{s}) \times \frac{(1-x_R)^{(2n_{spectators}-1)}}{(p_T^2)^{(n_{participants}-2)}}$$



helicity
conservation

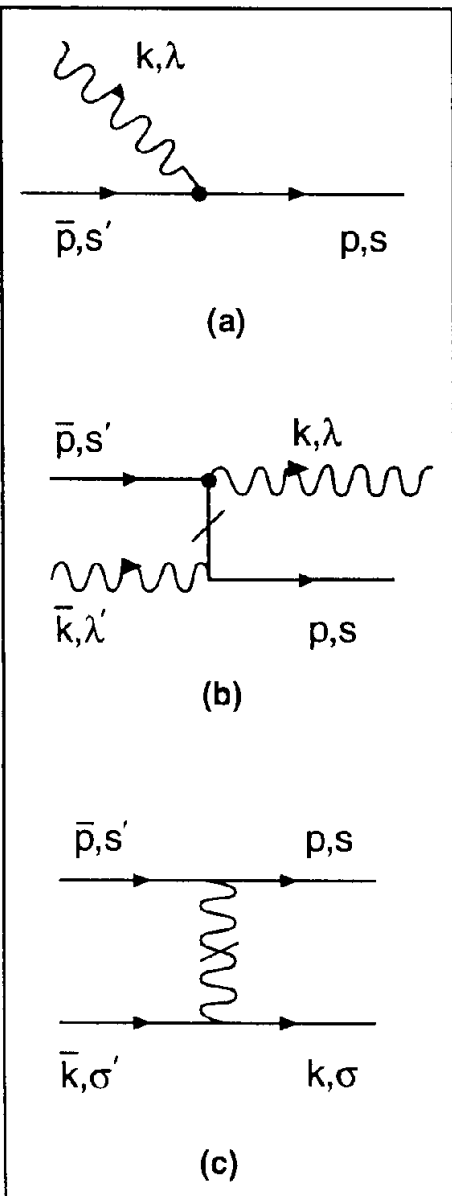
Farrar, Jackson;
Lepage, sjb;
Burkardt,
Schmidt, Sjb

Light-Front QCD Heisenberg Equation

$$H_{LC}^{QCD} |\Psi_h\rangle = \mathcal{M}_h^2 |\Psi_h\rangle$$

DLCQ: Solved QCD(1+1) for any quark mass and flavors

Hornbostel, Pauli, sjb



n	Sector	1 q \bar{q}	2 gg	3 q \bar{q} g	4 q \bar{q} q \bar{q}	5 gg g	6 q \bar{q} gg	7 q \bar{q} q \bar{q} g	8 q \bar{q} q \bar{q} q \bar{q}	9 gg gg	10 q \bar{q} gg g	11 q \bar{q} q \bar{q} gg	12 q \bar{q} q \bar{q} q \bar{q} g	13 q \bar{q} q \bar{q} q \bar{q} q \bar{q}
1	q \bar{q}				
2	gg			
3	q \bar{q} g							
4	q \bar{q} q \bar{q}	
5	gg g
6	q \bar{q} gg								.				.	.
7	q \bar{q} q \bar{q} g
8	q \bar{q} q \bar{q} q \bar{q}			
9	gg gg
10	q \bar{q} gg g
11	q \bar{q} q \bar{q} gg
12	q \bar{q} q \bar{q} q \bar{q} g			
13	q \bar{q} q \bar{q} q \bar{q} q \bar{q}		



Minkowski space; frame-independent; no fermion doubling; no ghosts

Discretized LF Quantization

DLCQ: Diagonalize QCD Hamiltonian, periodic LF BC

BLFQ (Vary et al)
Use LF Holographic Basis

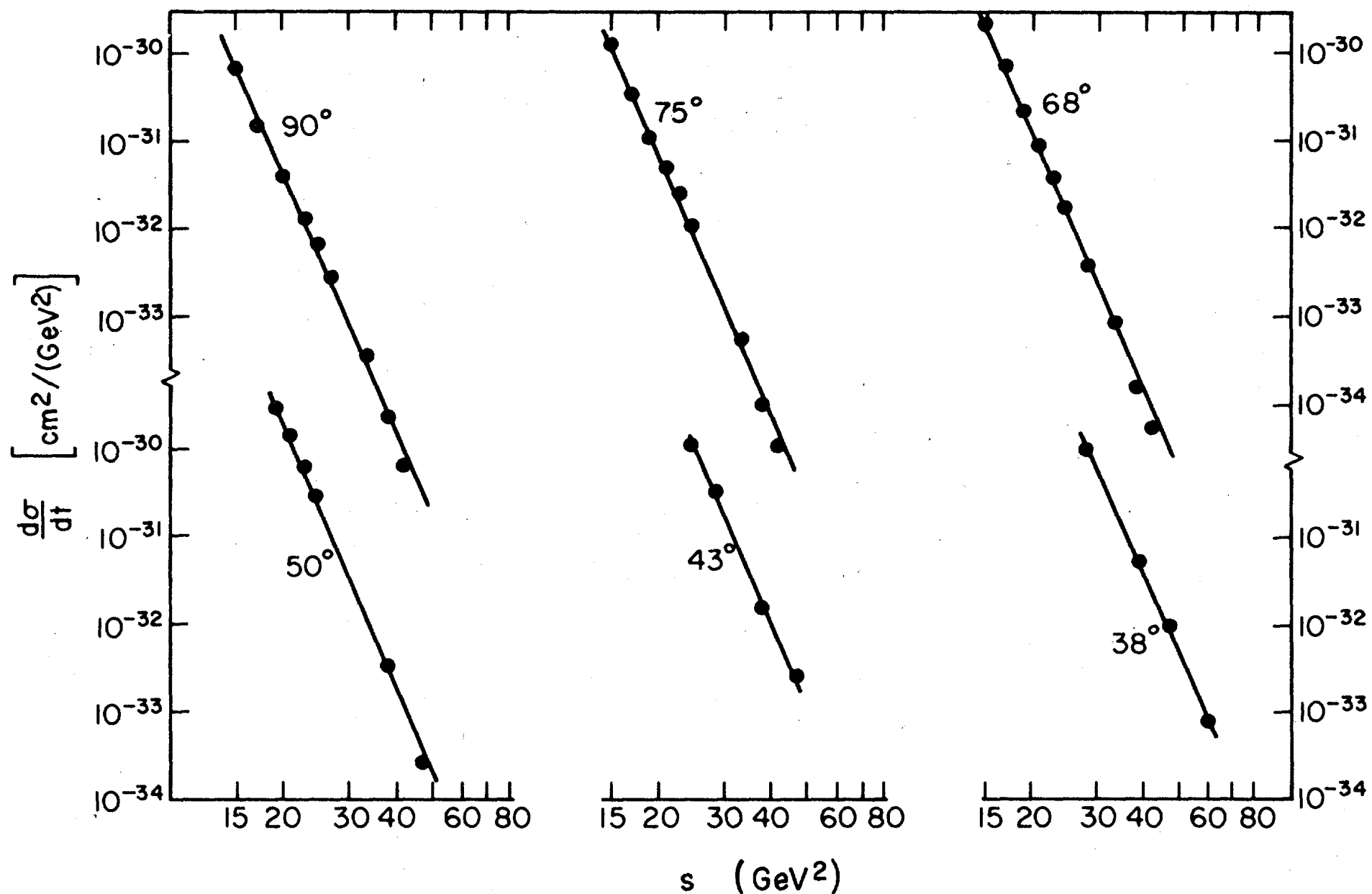
Solve QCD by Matrix Diagonalization

Diagonalize the LF Hamiltonian on an Orthonormal Basis
Lorentz Frame-Independent,
Minkowski Causal LF Time
Compute Hadron masses, LF Wavefunctions
Successful applications to QCD(1+1)
Use advanced computer resources
Competitive with LGTh?

H. C. Pauli, K. Hornbostel, sjb

Scaling of Hard Exclusive reactions: Fixed t/s

EXCLUSIVE PROCESSES IN PERTURBATIVE QUANTUM...



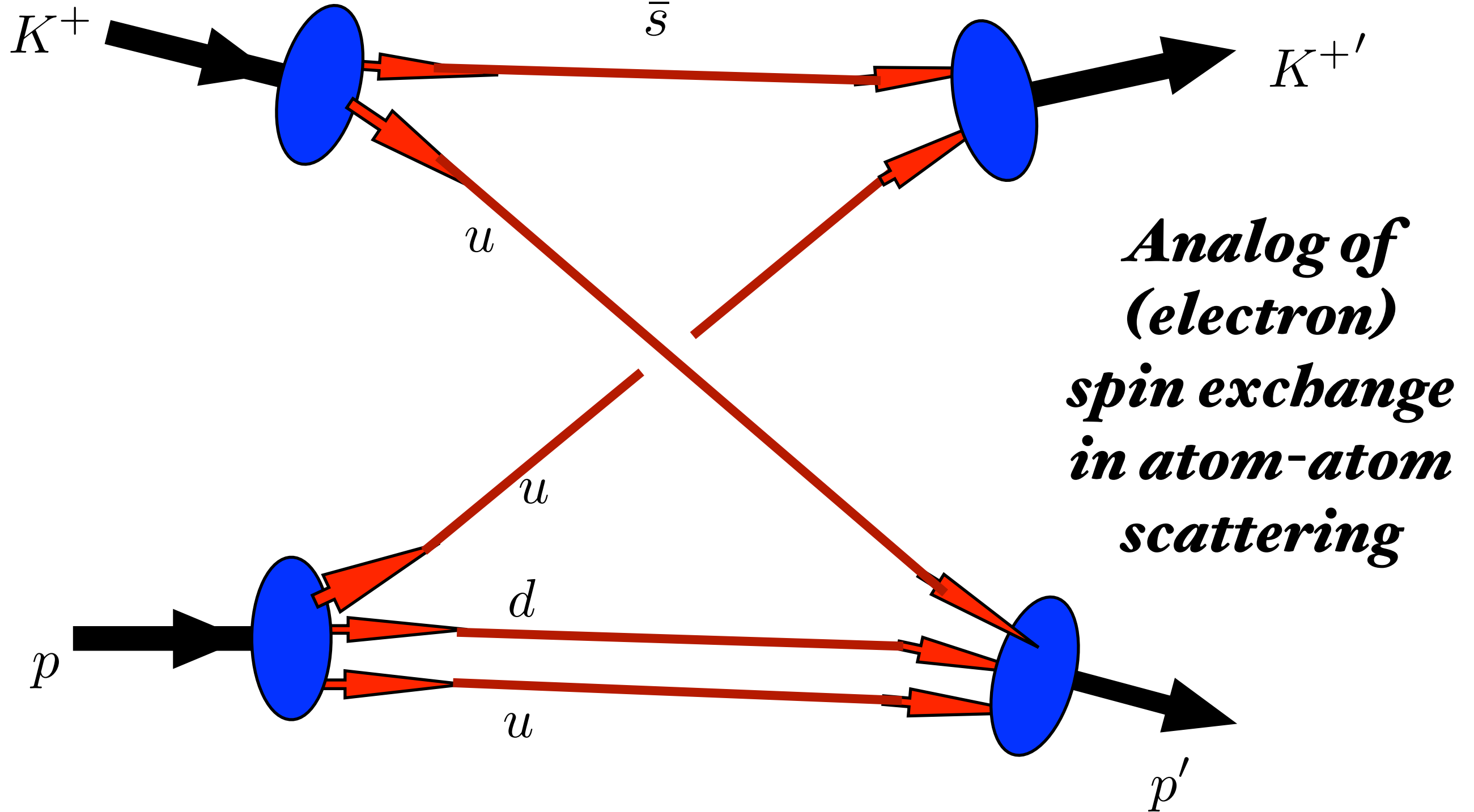
Cross sections for $pp \rightarrow pp$ at wide angles

The straight lines correspond to a falloff of $1/s^{10}$.

$$\frac{d\sigma}{dt}(p + p \rightarrow p + p) = \frac{F(\theta_{CM})}{s^{10}}$$

Manifestation of Asymptotic Freedom

$$K^+ p \rightarrow K^+ p$$

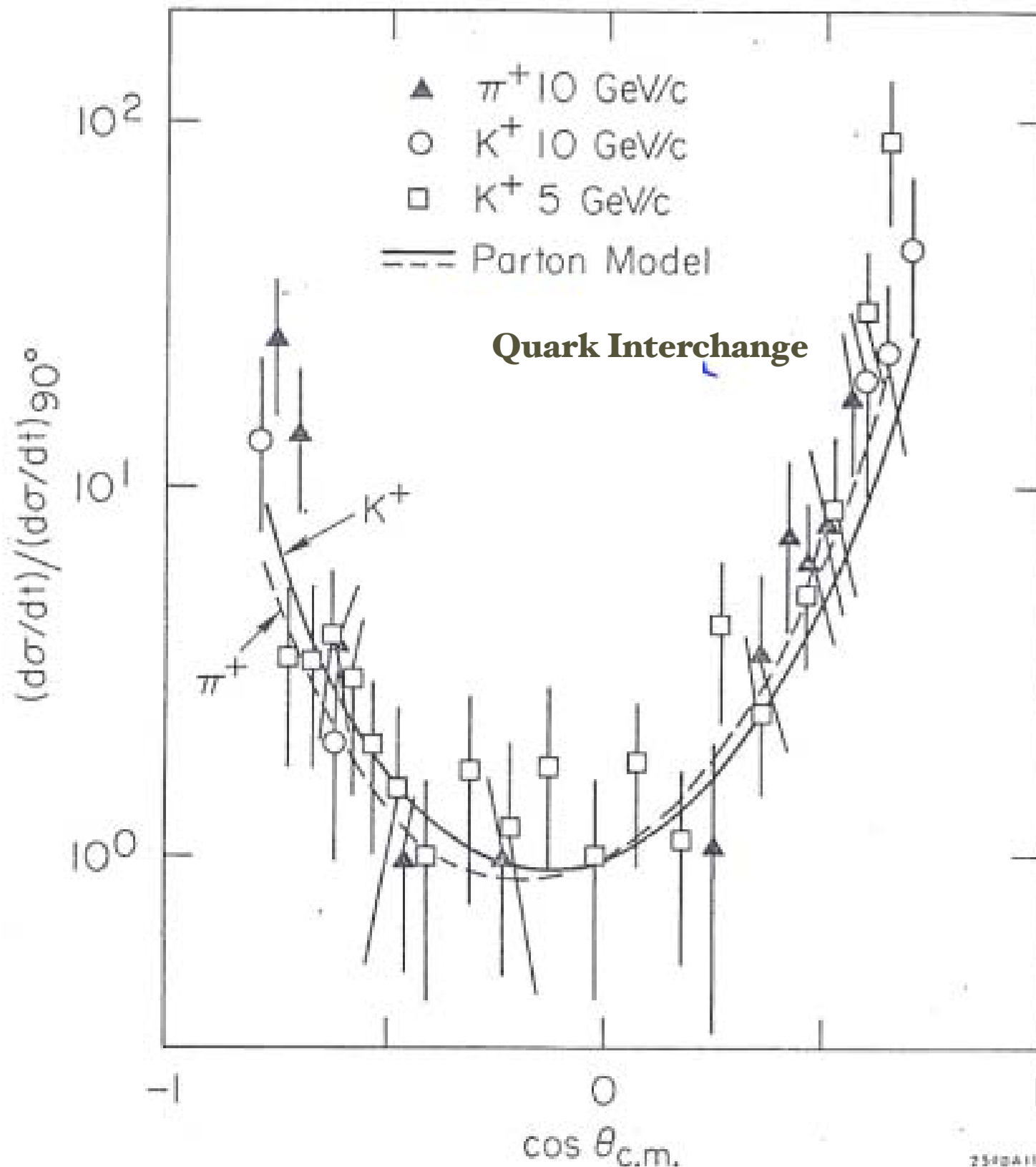


Quark Interchange

Blankenbecler, Gunion, sjb

Interactions between exchanged quarks suppressed at high momentum transfer

Quark Interchange Blankenbecler, Gunion, sjb



$$M(t, u)_{\text{interchange}} \propto \frac{1}{ut^2}$$

$$\frac{d\sigma}{dt}(K^+ p \rightarrow K^+ p) = \frac{F(t/s)}{s^8}$$

Non-linear Regge behavior:

$$\alpha_R(t) \rightarrow -1$$

$$\frac{d\sigma}{dt} = \frac{f(t/s)}{s^{N-2}} \quad N-2 = \# \text{ fundamental constituents} - 2 = 2+3+2+3-2=8$$

“Counting Rules” Farrar and sjb; Muradyan, Matveev, Tavkelidze

Challenge: Compute Hadron Structure, Spectroscopy, and Dynamics from QCD!

- **Color Confinement**
- **Origin of the QCD Mass Scale**
- **Meson and Baryon Spectroscopy**
- **Exotic States: Tetraquarks, Pentaquarks, Gluonium,**
- **Universal Regge Slopes: n , L , Mesons and Baryons**
- **Almost Massless Pion: GMOR Chiral Symmetry Breaking**
$$M_\pi^2 f_\pi^2 = -\frac{1}{2}(m_u + m_d) \langle \bar{u}u + \bar{d}d \rangle + \mathcal{O}((m_u + m_d)^2)$$
- **QCD Coupling at all Scales $\alpha_s(Q^2)$**
- **Eliminate Scale Uncertainties and Scheme Dependence: BLM/PMC (Principle of Maximum Conformality)**

Need a First Approximation to QCD

*Comparable in simplicity to
Schrödinger Theory in Atomic Physics*

Relativistic, Frame-Independent, Color-Confining

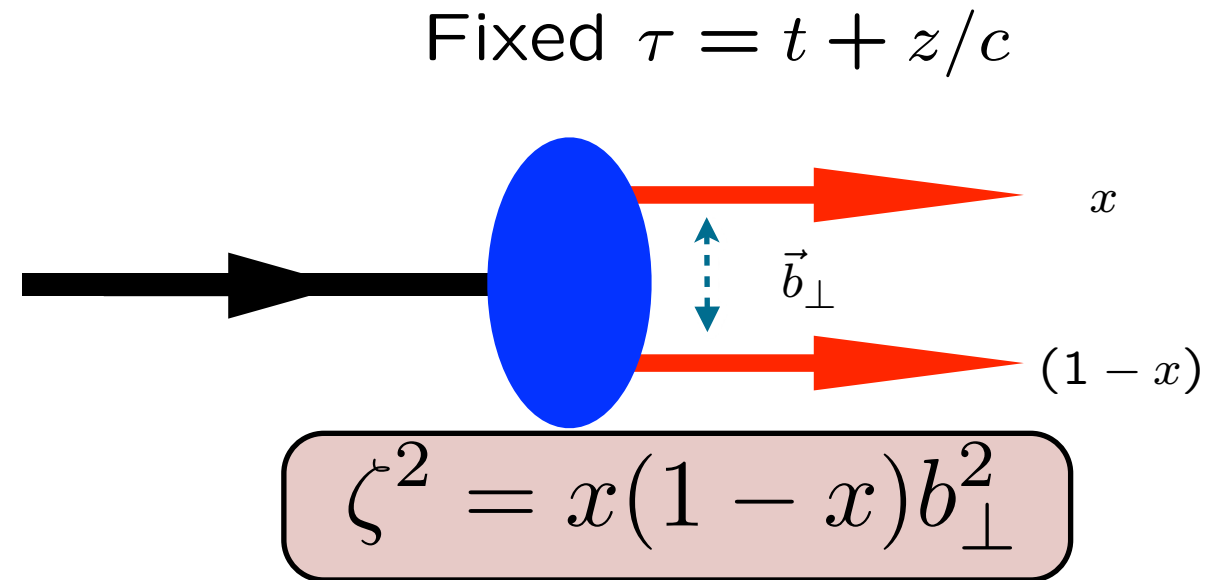
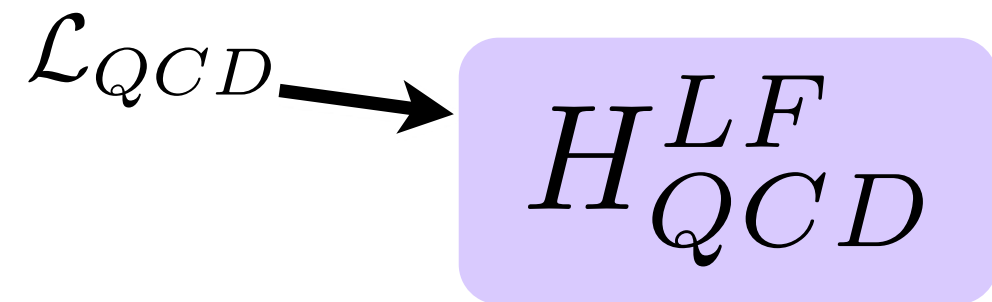
Origin of hadronic mass scale if $m_q=0$

Semi-Classical Approximation to QCD

de Téramond, Dosch, Lorcé, sjb

**AdS/QCD
Light-Front Holography**

Light-Front QCD



$$(H_{LF}^0 + H_{LF}^I) |\Psi\rangle = M^2 |\Psi\rangle$$

Coupled Fock states

*Eliminate higher Fock states
and retarded interactions*

$$\left[\frac{\vec{k}_{\perp}^2 + m^2}{x(1-x)} + V_{\text{eff}}^{LF} \right] \psi_{LF}(x, \vec{k}_{\perp}) = M^2 \psi_{LF}(x, \vec{k}_{\perp})$$

Effective two-particle equation

$$\left[-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = \mathcal{M}^2 \psi(\zeta)$$

Azimuthal Basis ζ, ϕ

Single variable Equation

$$m_q = 0$$

*Confining AdS/QCD
potential!*

AdS/QCD: LF Holography

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

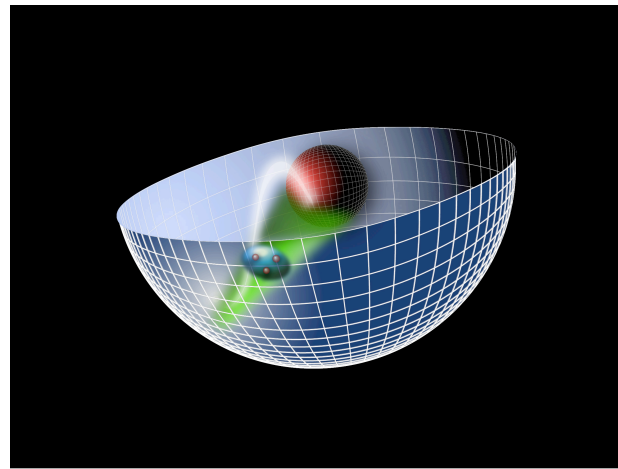
Sums an infinite # diagrams

Semiclassical first approximation to QCD

de Téramond, Dosch, Lorcé, sjb

*AdS/QCD
Soft-Wall Model*

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)b_{\perp}^2$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = M^2 \psi(\zeta)$$



Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

Single variable ζ

***Unique
Confinement Potential!***

*Conformal Symmetry
of the AdS action*

Confinement scale:

$$\kappa \simeq 0.5 \text{ GeV}$$

● de Alfaro, Fubini, Furlan:

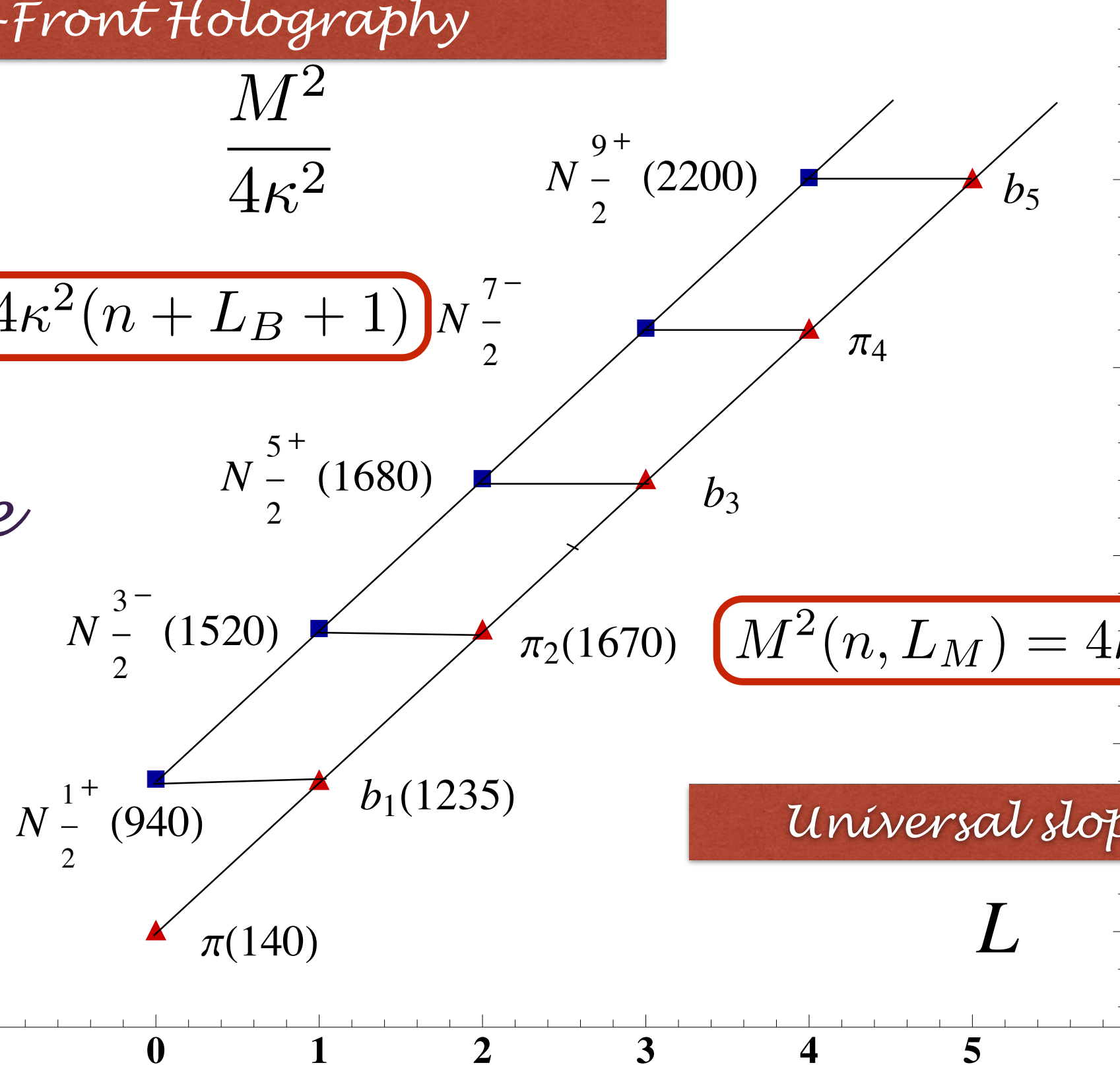
● Fubini, Rabinovici:

***Scale can appear in Hamiltonian and EQM
without affecting conformal invariance of AdS action!***

GeV units external to QCD: Ratios of Masses Determined

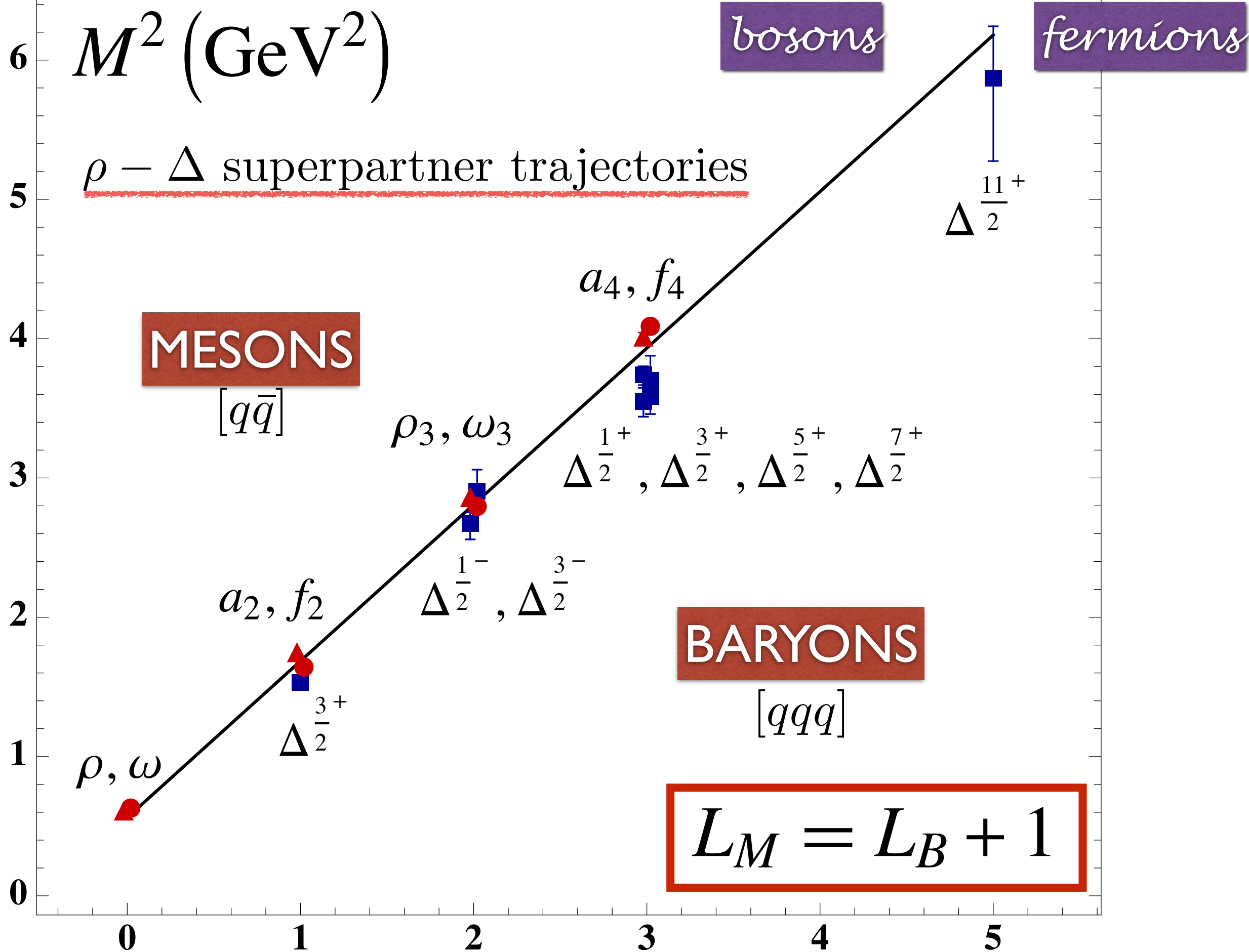
$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

Same slope



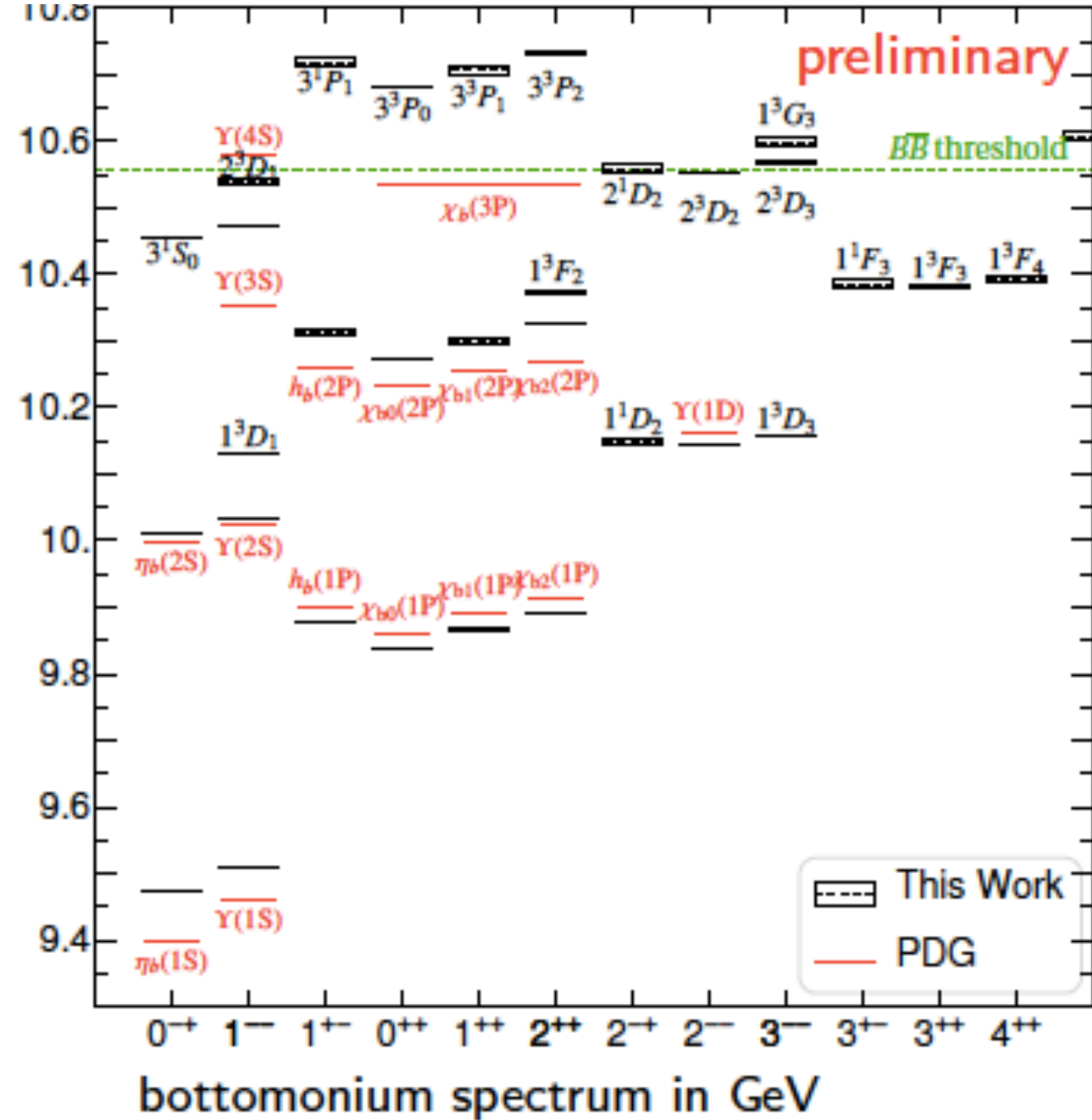
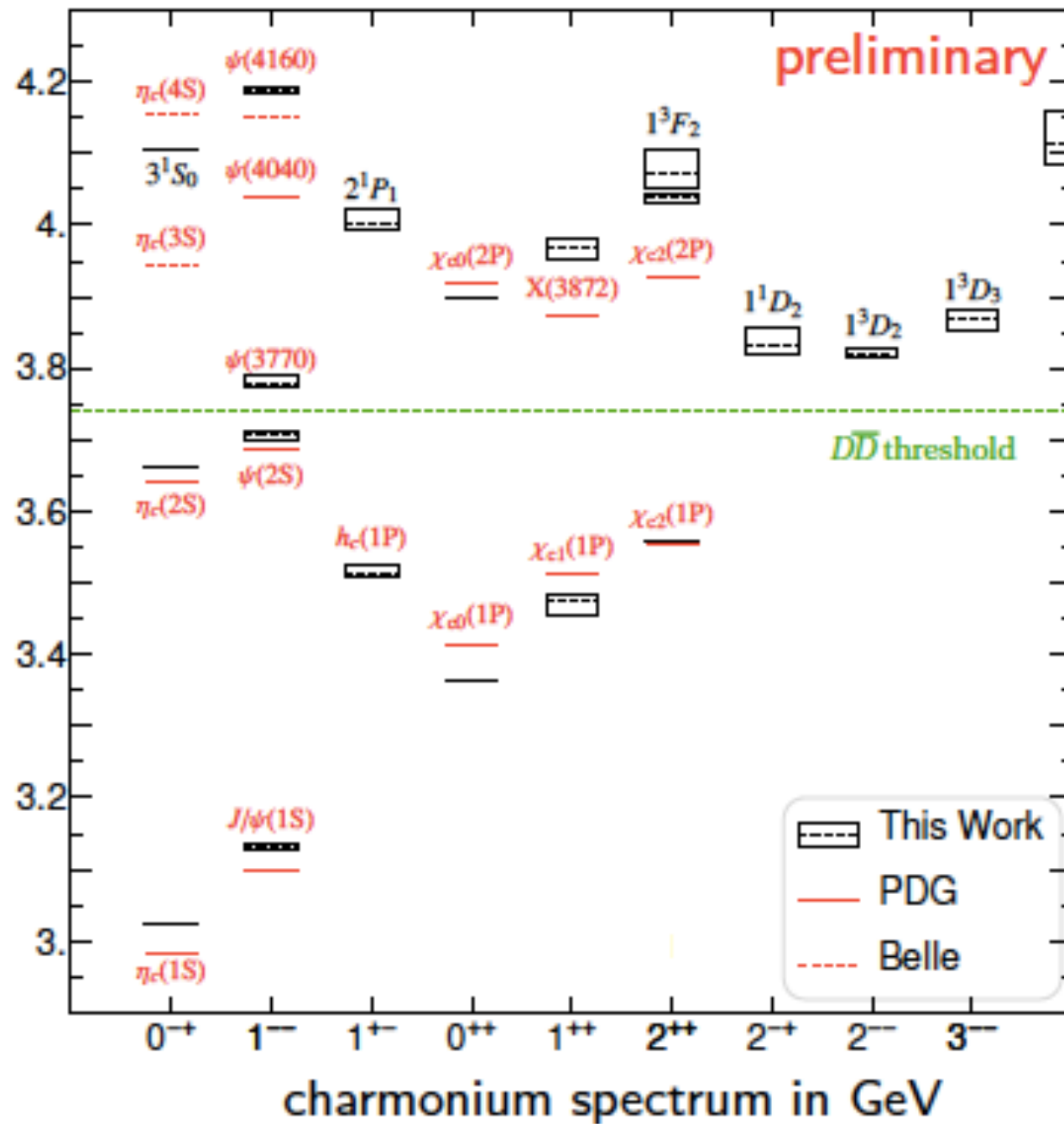
$$\frac{M_{meson}^2}{M_{nucleon}^2} = \frac{n + L_M}{n + L_B + 1}$$

**Meson-Baryon
Mass Degeneracy
for $L_M = L_B + 1$**



Heavy Quarkonium in a Light-Front Holographic Basis

BLFQ using AdS/QCD

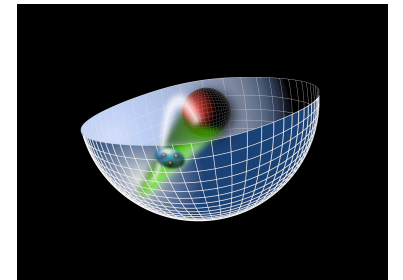


Yang Li , Pieter Maris Xingbo Zhao, James P. Vary PLB 758, 116 (2016)

$$H_{\text{eff}} = \underbrace{\frac{\vec{k}_{\perp}^2 + m_q^2}{x} + \frac{\vec{k}_{\perp}^2 + m_{\bar{q}}^2}{1-x}}_{\text{LF kinetic energy}} + \underbrace{\kappa^4 \zeta_{\perp}^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2} \partial_x [x(1-x) \partial_x]}_{\text{confinement}} - \underbrace{\frac{C_F 4\pi\alpha_s}{Q^2} \bar{u}_{s'}(k') \gamma_{\mu} u_s(k) \bar{v}_{\bar{s}}(\bar{k}) \gamma^{\mu} v_{\bar{s}'}(\bar{k}')}_{\text{one-gluon exchange}}$$

Dilaton-Modified AdS

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2} (\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$



- **Soft-wall dilaton profile breaks**

- **Color Confinement in z**

- **Introduces confinement scale κ**

- **Uses AdS_5 as template for conformal theory**

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

AdS/CFT

D. Gross: duality of QCD with string theory

Introduce “Dilaton” to simulate confinement analytically

- Nonconformal metric dual to a confining gauge theory

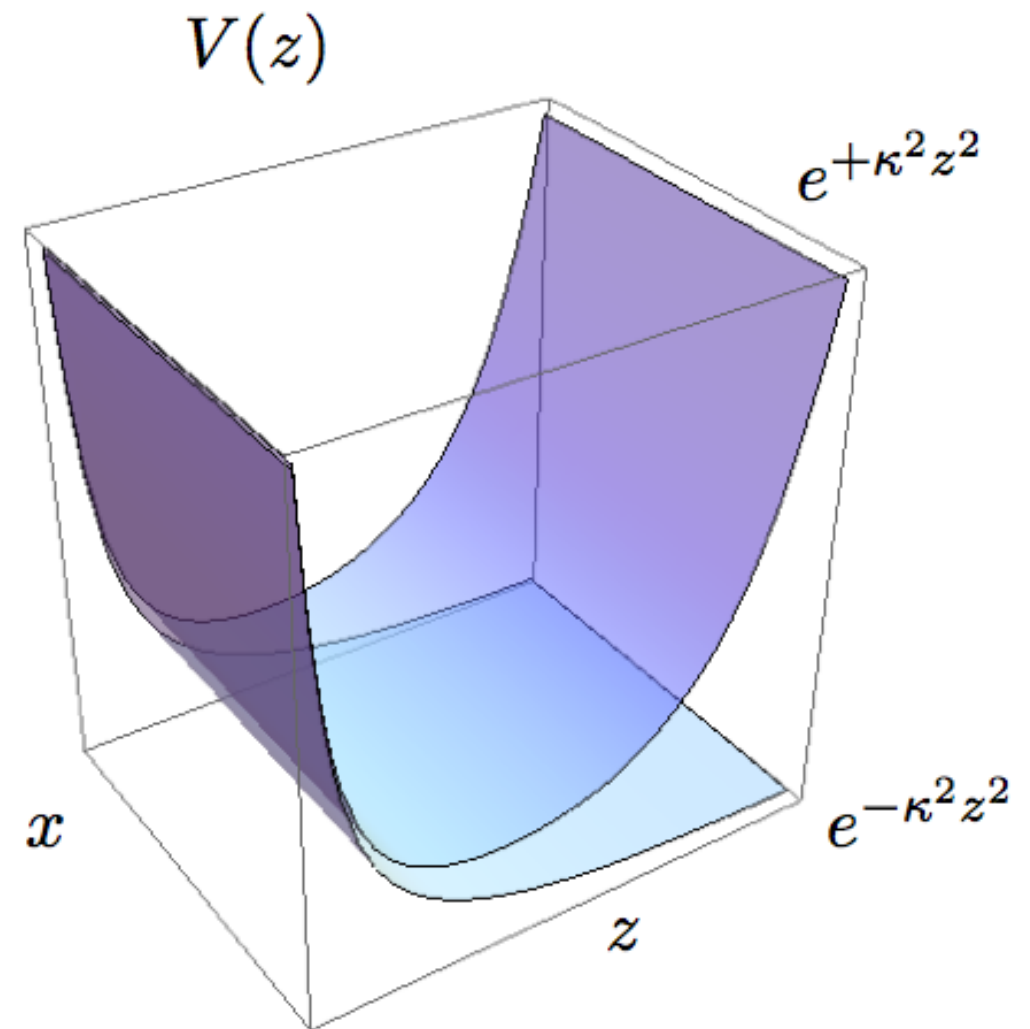
$$ds^2 = \frac{R^2}{z^2} e^{\varphi(z)} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2)$$

where $\varphi(z) \rightarrow 0$ at small z for geometries which are asymptotically AdS_5

- Gravitational potential energy for object of mass m

$$V = mc^2 \sqrt{g_{00}} = mc^2 R \frac{e^{\varphi(z)/2}}{z}$$

- Consider warp factor $\exp(\pm \kappa^2 z^2)$
- Plus solution: $V(z)$ increases exponentially confining any object in modified AdS metrics to distances $\langle z \rangle \sim 1/\kappa$



Klebanov and Maldacena

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

- de Te'ramond, sjb

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

• de Teramond, sjb

AdS Soft-Wall Schrödinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1 - 4L^2}{4z^2} + U(z) \right] \Phi(z) = \mathcal{M}^2 \Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2 (L + S - 1)$$

Derived from variation of Action for Dilaton-Modified AdS_5

Identical to Single-Variable Light-Front Bound State Equation in ζ !

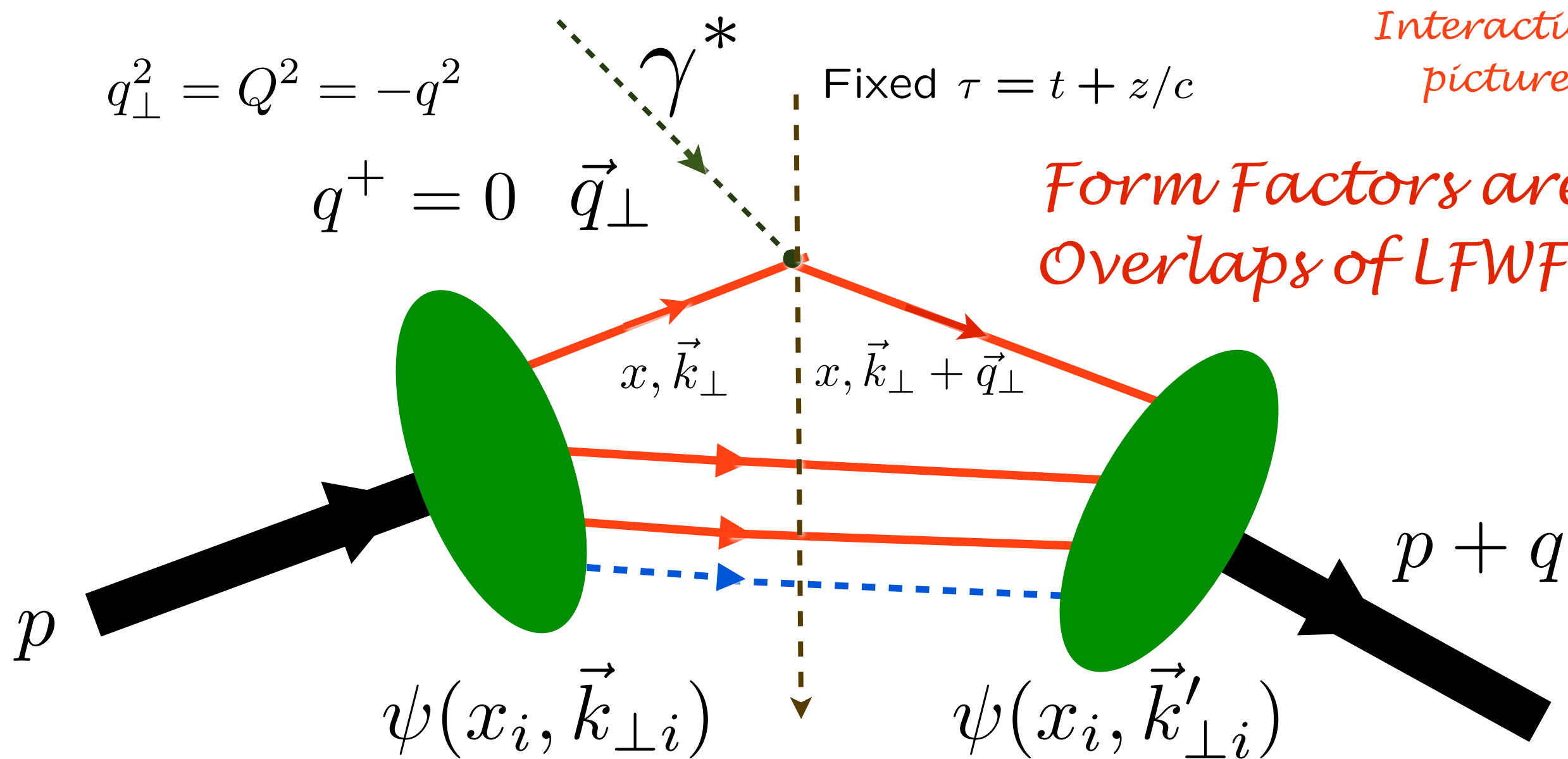
$$z \quad \longleftrightarrow \quad \zeta = \sqrt{x(1-x)} \vec{b}_\perp^2$$

Light-Front Holography

$$\langle p+q | j^+(0) | p \rangle = 2p^+ F(q^2)$$

Front Form

Interaction
picture



struck $\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i) \vec{q}_{\perp}$

spectators $\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i \vec{q}_{\perp}$

Drell & Yan, West
Exact LF formula!

Drell, sjb

Transverse size $\propto \frac{1}{Q}$

Holographic Mapping of AdS Modes to QCD LFWFs

*Drell-Yan-West: Form Factors are
Convolution of LFWFs*

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \frac{(1-x)}{x} \int \zeta d\zeta J_0 \left(\zeta q \sqrt{\frac{1-x}{x}} \right) \tilde{\rho}(x, \zeta),$$

with $\tilde{\rho}(x, \zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\zeta = \sqrt{x(1-x)} \vec{b}_\perp^2$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0 \left(\zeta Q \sqrt{\frac{1-x}{x}} \right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q, \zeta) = \zeta Q K_1(\zeta Q)$!

de Te'ramond, sjb

Identical to Polchinski-Strassler Convolution of AdS Amplitudes

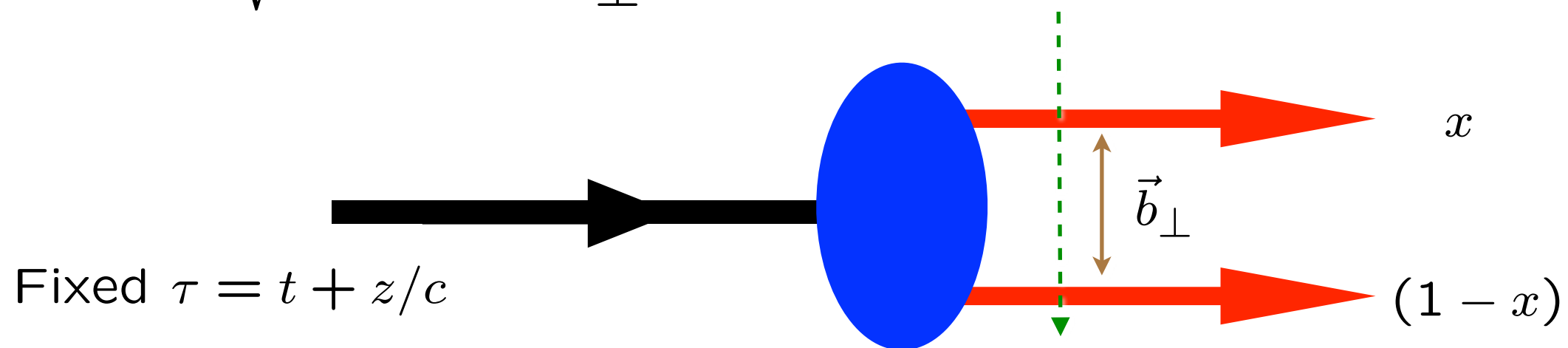
LF(3+1) \longleftrightarrow AdS₅

$$LF(3+1) \longleftrightarrow AdS_5$$

Light-Front Holographic Dictionary

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)} \vec{b}_\perp^2 \longleftrightarrow z$$



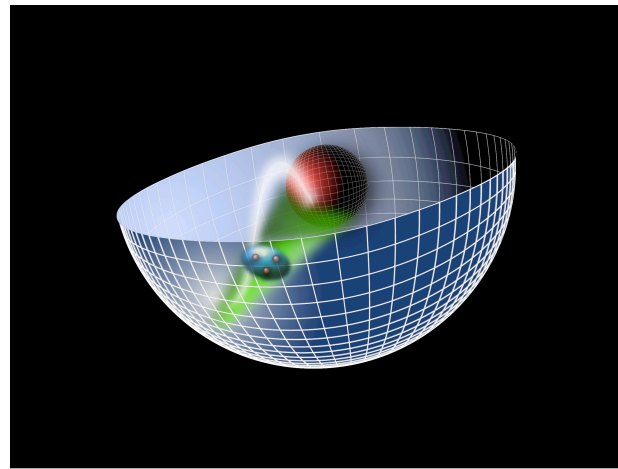
$$\psi(x, \zeta) = \sqrt{x(1-x)} \zeta^{-1/2} \phi(\zeta)$$

$$(\mu R)^2 = L^2 - (J - 2)^2$$

Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion

*AdS/QCD
Soft-Wall Model*

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)b_{\perp}^2$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta) \right] \psi(\zeta) = M^2 \psi(\zeta)$$



Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)$$

Single variable ζ

***Unique
Confinement Potential!***

*Conformal Symmetry
of the action*

Confinement scale:

$$\kappa \simeq 0.5 \text{ GeV}$$

- de Alfaro, Fubini, Furlan:
- Fubini, Rabinovici:

***Scale can appear in Hamiltonian and EQM
without affecting conformal invariance of action!***

GeV units external to QCD: Ratios of Masses Determined

$$\left(-\partial_{\zeta}^2 + \kappa^4 \zeta^2 + 2\kappa^2(L_B + 1) + \frac{4L_B^2 - 1}{4\zeta^2} \right) \psi_J^+ = M^2 \psi_J^+ \quad \left. \vphantom{\left(-\partial_{\zeta}^2 + \kappa^4 \zeta^2 + 2\kappa^2(L_B + 1) + \frac{4L_B^2 - 1}{4\zeta^2} \right) \psi_J^+ = M^2 \psi_J^+} \right\}$$

$$\left(-\partial_{\zeta}^2 + \kappa^4 \zeta^2 + 2\kappa^2 L_B + \frac{4(L_B + 1)^2 - 1}{4\zeta^2} \right) \psi_J^- = M^2 \psi_J^- \quad \left. \vphantom{\left(-\partial_{\zeta}^2 + \kappa^4 \zeta^2 + 2\kappa^2 L_B + \frac{4(L_B + 1)^2 - 1}{4\zeta^2} \right) \psi_J^- = M^2 \psi_J^-} \right\}$$

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1) \quad \mathbf{S=1/2, P=+}$$

Meson Equation

$$\lambda = \kappa^2$$

$$\left(-\partial_{\zeta}^2 + \kappa^4 \zeta^2 + 2\kappa^2(J - 1) + \frac{4L_M^2 - 1}{4\zeta^2} \right) \phi_J = M^2 \phi_J$$

$$M^2(n, L_M) = 4\kappa^2(n + L_M) \quad \mathbf{S=0, P=+}$$

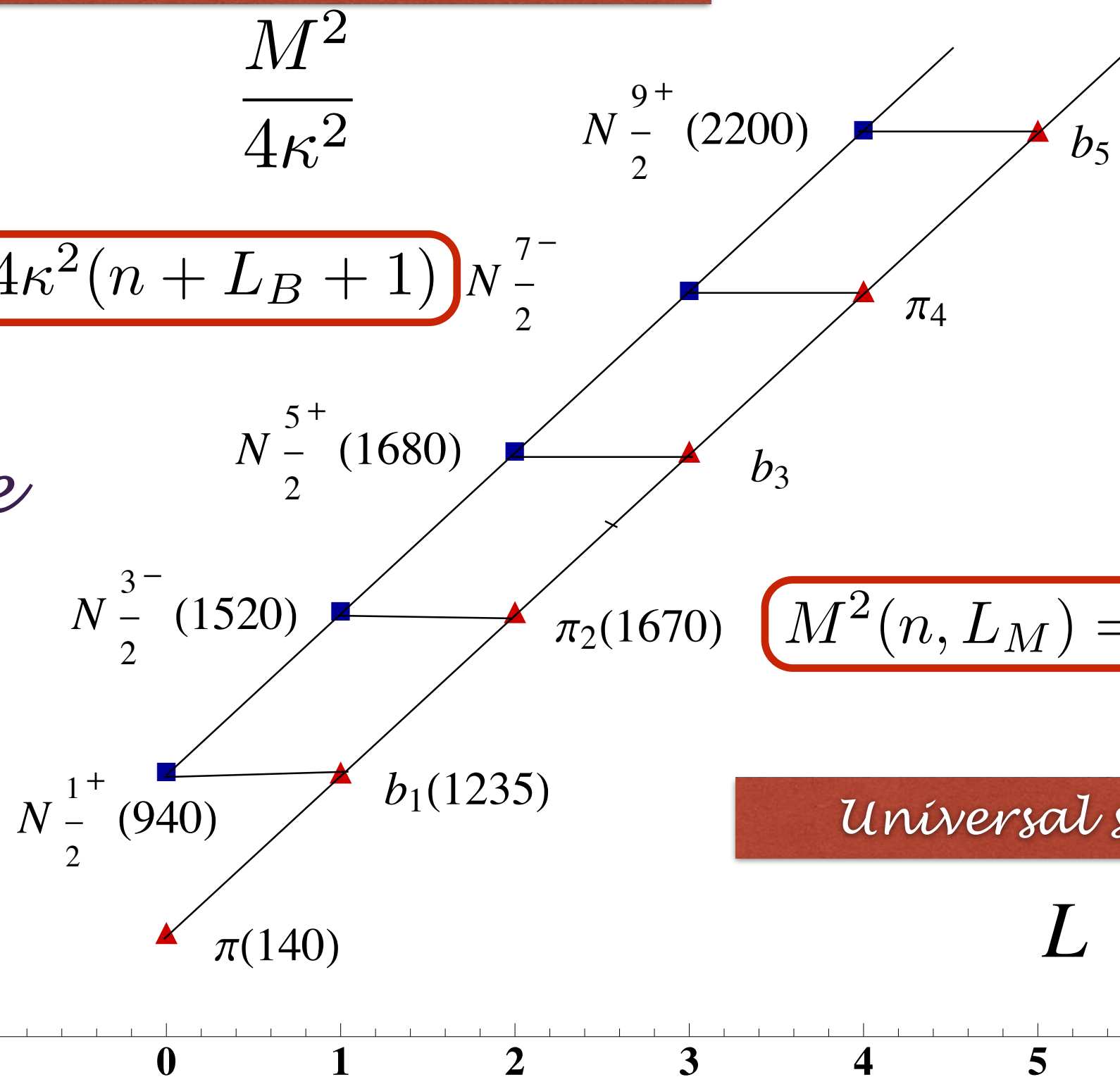
Same κ !

$S=0, I=I$ Meson is superpartner of $S=1/2, I=I$ Baryon

Meson-Baryon Degeneracy for $L_M=L_B+1$

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

Same slope



$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Universal slopes in n, L

$$\frac{M_{meson}^2}{M_{nucleon}^2} = \frac{n + L_M}{n + L_B + 1}$$

**Meson-Baryon
Mass Degeneracy
for $L_M = L_B + 1$**

Massless pion!

Meson Spectrum in Soft Wall Model

$$m_\pi = 0 \text{ if } m_q = 0$$

Pion: Negative term for $J=0$ cancels positive terms from LFKÉ and potential



- Effective potential: $U(\zeta^2) = \kappa^4 \zeta^2 + 2\kappa^2(J - 1)$

- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2(J - 1) \right) \phi_J(\zeta) = M^2 \phi_J(\zeta)$$

- Normalized eigenfunctions $\langle \phi | \phi \rangle = \int d\zeta \phi^2(z)^2 = 1$

$$\phi_{n,L}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^L(\kappa^2 \zeta^2)$$

- Eigenvalues

$$\mathcal{M}_{n,J,L}^2 = 4\kappa^2 \left(n + \frac{J+L}{2} \right)$$

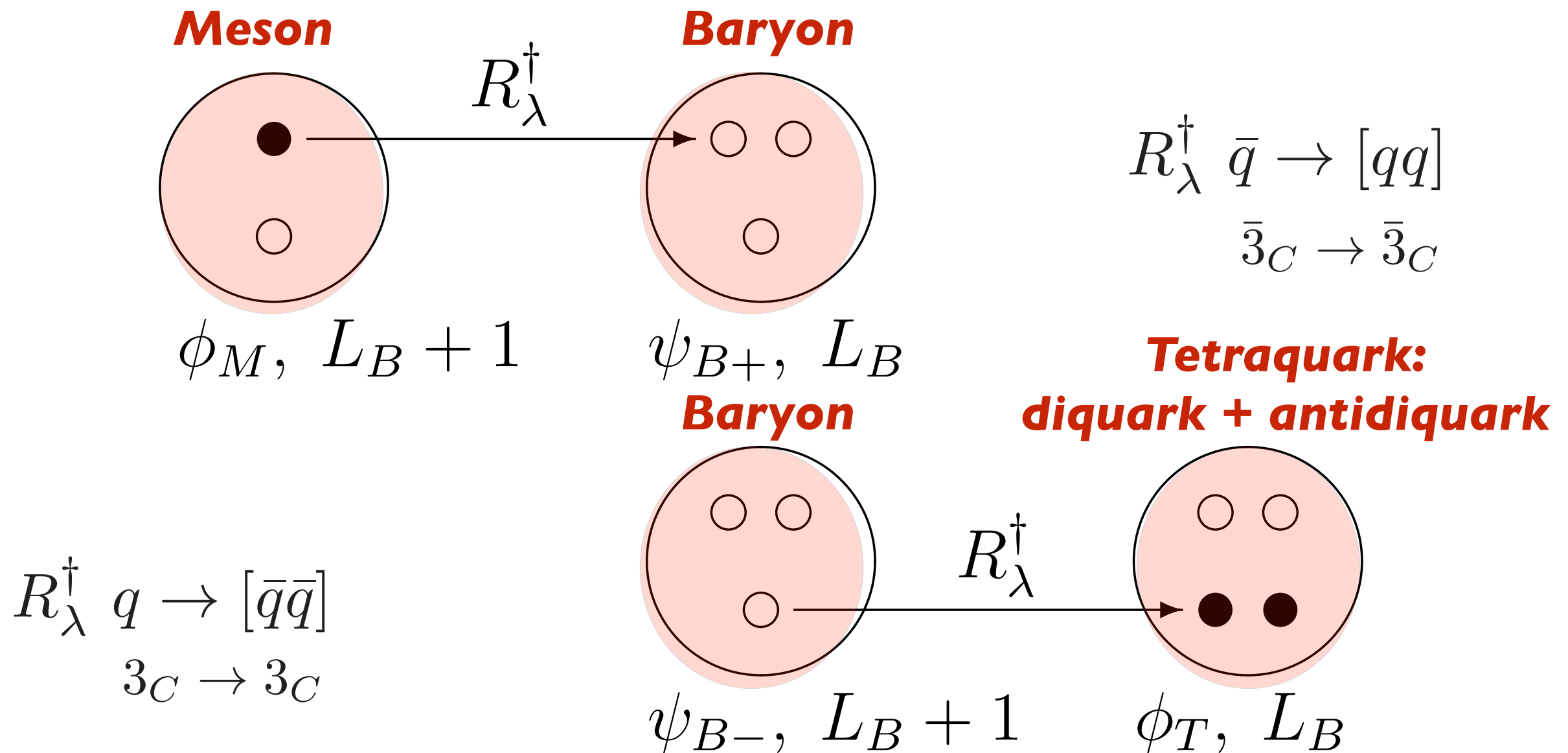
$$\vec{\zeta}^2 = \vec{b}_\perp^2 x(1-x)$$

G. de Te'ramond, H. G. Dosch, sjb

Superconformal Algebra

Four-Plet Representations

Bosons, Fermions with Equal Mass!



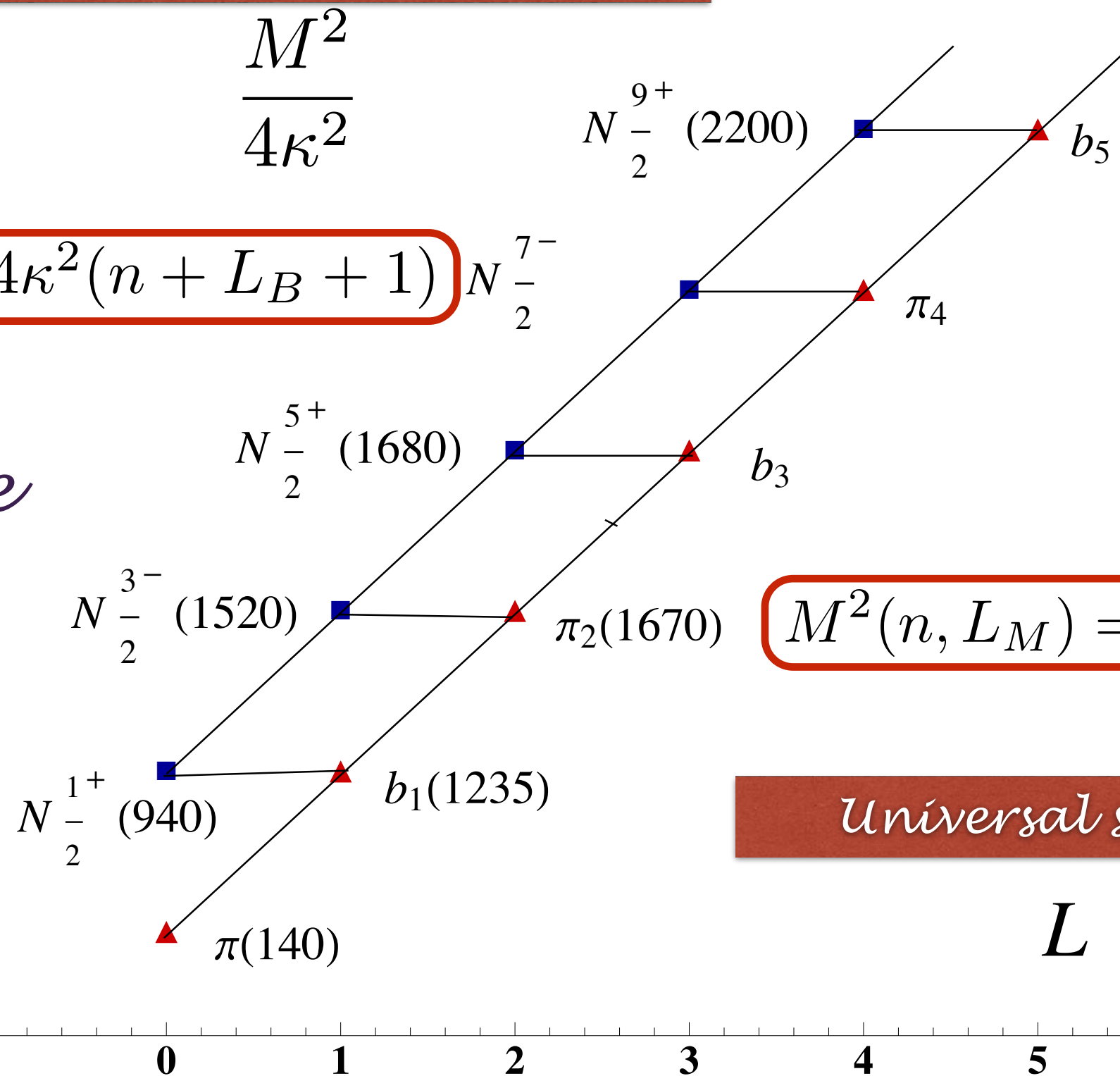
Proton: $|u[ud]\rangle$ Quark + Scalar Diquark
Equal Weight: $L=0, L=1$

Superconformal Quantum Mechanics Light-Front Holography

de Téramond, Dosch, Lorcé, sjb

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

Same slope



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Universal slopes in n, L

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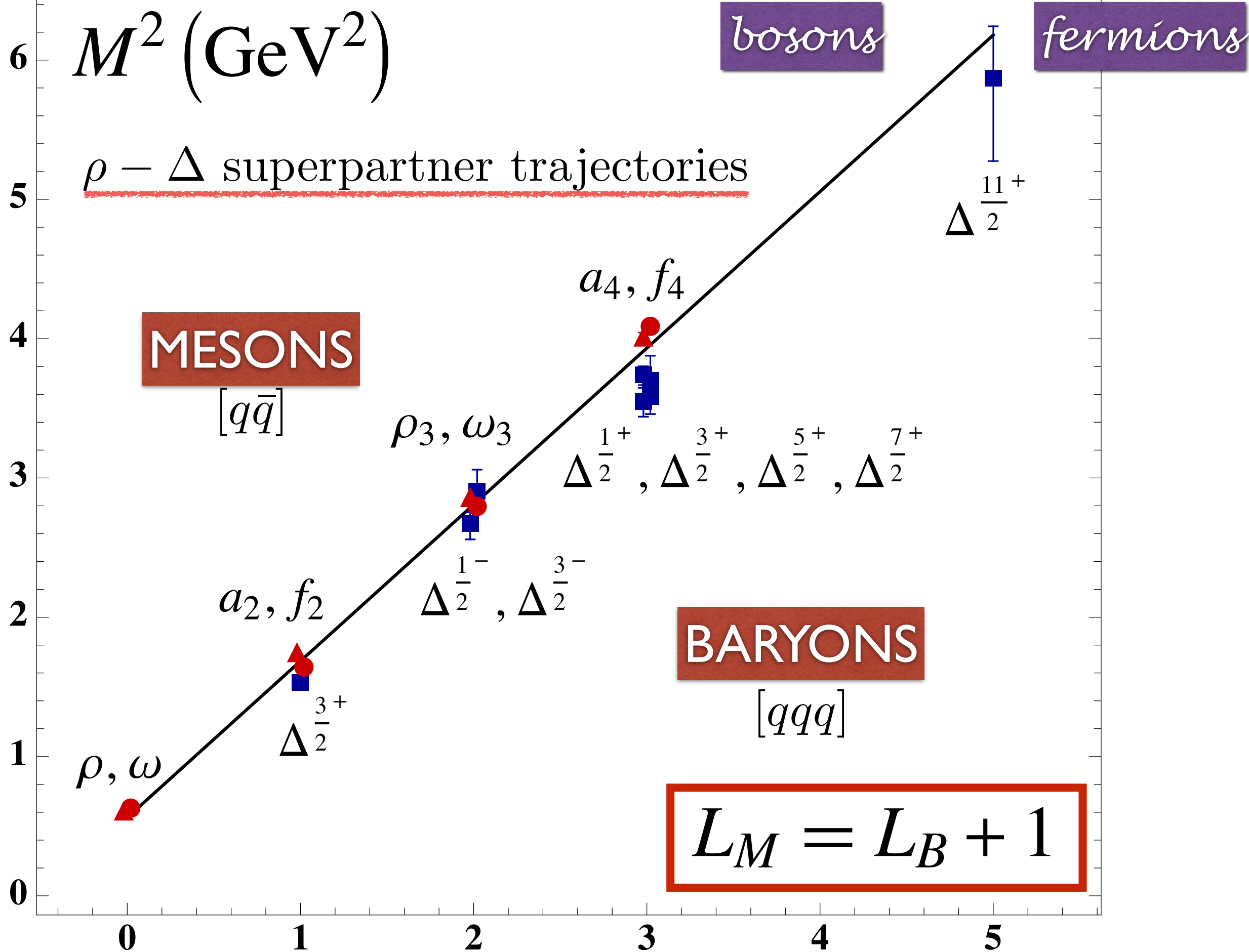
$$\phi_{n,L}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \zeta^{1/2+L} e^{-\kappa^2 \zeta^2 / 2} L_n^L(\kappa^2 \zeta^2)$$

- Eigenvalues

$$\mathcal{M}_{n,J,L}^2 = 4\kappa^2 \left(n + \frac{J+L}{2} \right)$$

$$\vec{\zeta}^2 = \vec{b}_\perp^2 x(1-x)$$

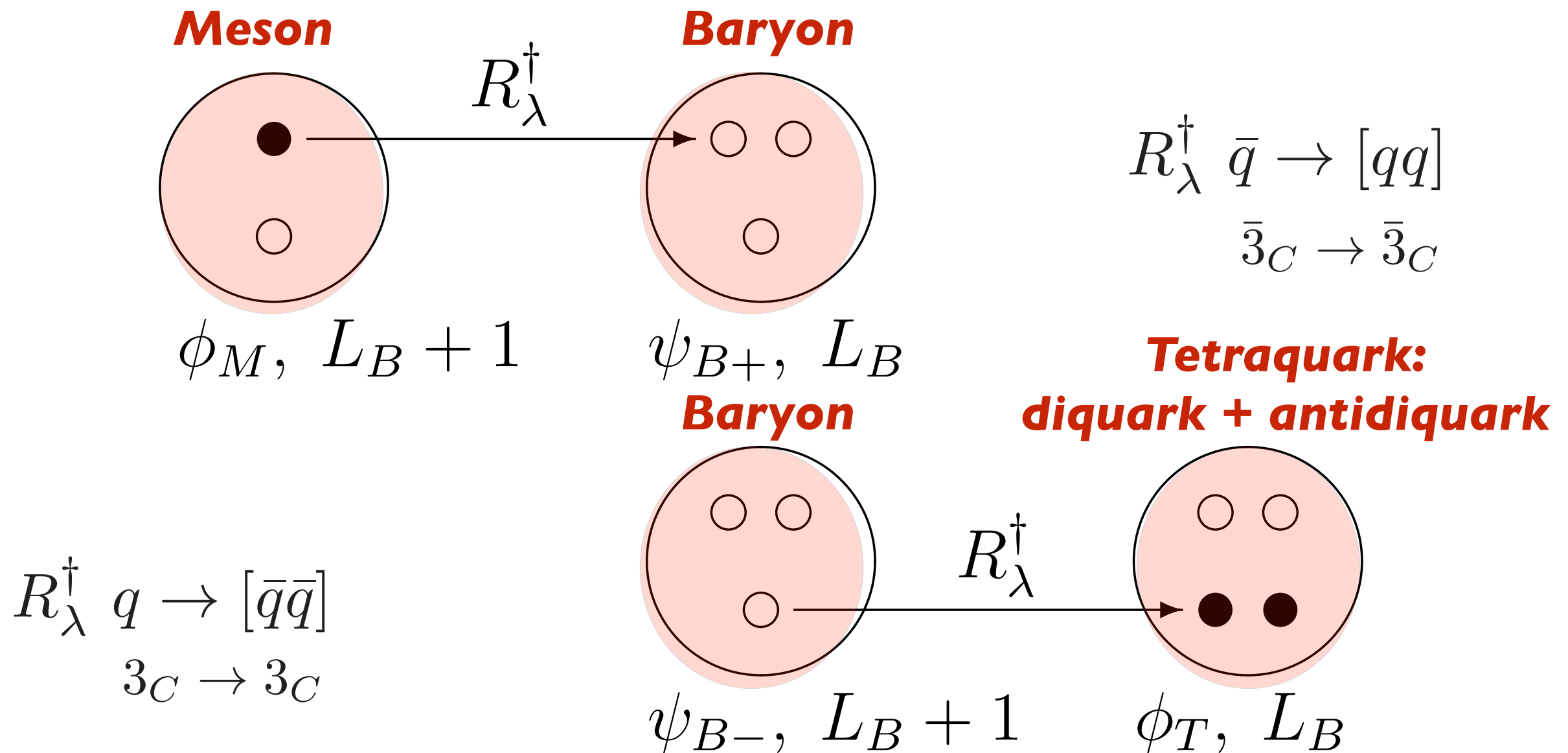
G. de Teramond, H. G. Dosch, sjb



Superconformal Algebra

Four-Plet Representations

Bosons, Fermions with Equal Mass!



Proton: $|u[ud]\rangle$ Quark + Scalar Diquark
Equal Weight: $L=0, L=1$

Universal Hadronic Decomposition

$$\frac{\mathcal{M}_H^2}{\kappa^2} = (1 + 2n + L) + (1 + 2n + L) + (2L + 4S + 2B - 2)$$

- **Universal quark light-front kinetic energy**

**Equal:
Virial
Theorem**

$$\Delta \mathcal{M}_{LFKE}^2 = \kappa^2(1 + 2n + L)$$

- **Universal quark light-front potential energy**

$$\Delta \mathcal{M}_{LFPE}^2 = \kappa^2(1 + 2n + L)$$

- **Universal Constant Contribution from AdS and Superconformal Quantum Mechanics**

$$\Delta \mathcal{M}_{spin}^2 = 2\kappa^2(L + 2S + B - 1)$$

hyperfine spin-spin

Supersymmetry in QCD

- A hidden symmetry of Color $SU(3)_c$ in hadron physics:
- Relates meson and baryon spectroscopy
- QCD: No squarks or gluinos!
- Emerges from Light-Front Holography and Super-Conformal Algebra
- Color Confinement

de Téramond, Dosch, Lorcé, sjb

Input: one fundamental mass scale

$$\kappa = \sqrt{\lambda} = 0.523 \pm 0.024 \text{ GeV}$$

$M^2 \text{ (GeV}^2\text{)}$

$\rho - \Delta$ superpartner trajectories

bosons

fermions

6

5

4

3

2

1

0

MESONS

$[q\bar{q}]$

Supersymmetric
QCD Spectroscopy

a_4, f_4

ρ_3, ω_3

a_2, f_2

ρ, ω

BARYONS

$[qqq]$

$$L_M = L_B + 1$$

$\Delta_{\frac{1}{2}}^{1-}, \Delta_{\frac{3}{2}}^{3-}$

$\Delta_{\frac{1}{2}}^{1+}, \Delta_{\frac{3}{2}}^{3+}, \Delta_{\frac{5}{2}}^{5+}, \Delta_{\frac{7}{2}}^{7+}$

$\Delta_{\frac{11}{2}}^{11+}$

Remarkable Features of Light-Front Schrödinger Equation

Dynamics + Spectroscopy!

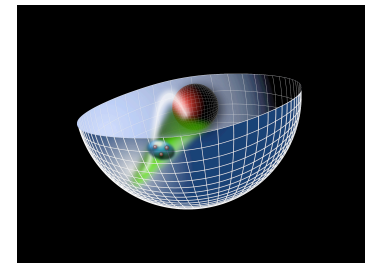
- **Relativistic, frame-independent**
- **QCD scale appears - unique LF potential**
- **Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter**
- **Zero-mass pion for zero mass quarks!**
- **Regge slope same for n and L -- not usual HO**
- **Splitting in L persists to high mass -- contradicts conventional wisdom based on breakdown of chiral symmetry**
- **Phenomenology: LFWFs, Form factors, electroproduction**
- **Extension to heavy quarks**

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$$

LFHQCD: Underlying Principles

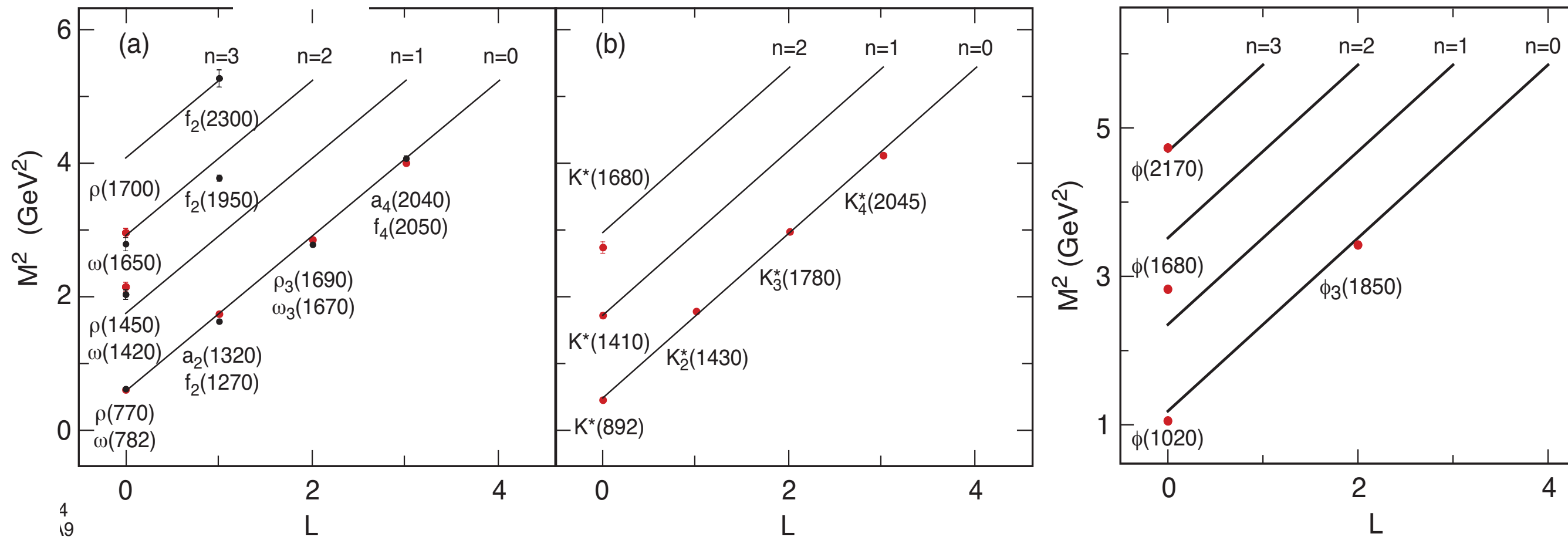
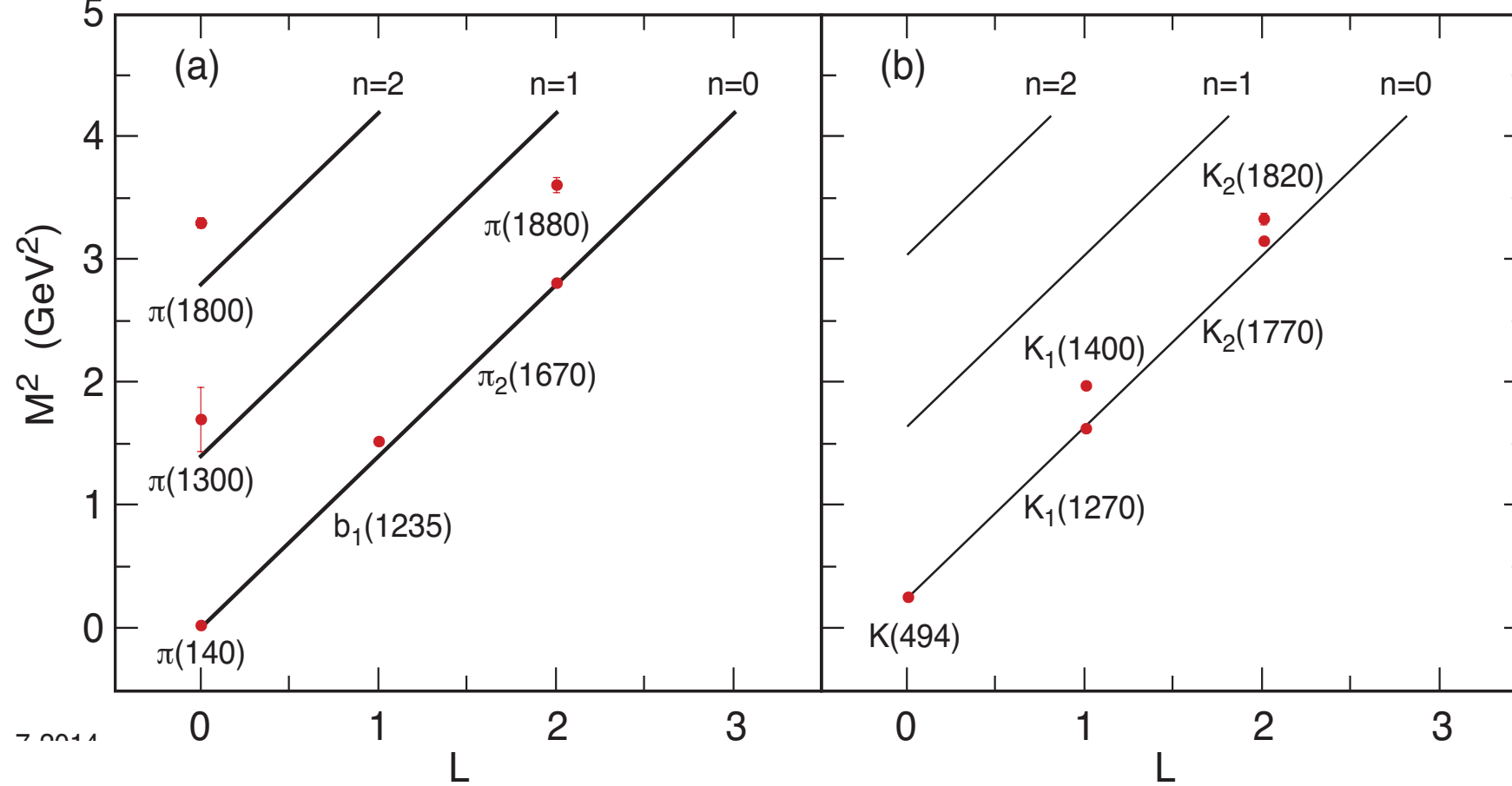
- **Poincaré Invariance: Independent of the observer's Lorentz frame: Quantization at Fixed Light-Front Time τ**
- **Causality: Information within causal horizon: Light-Front**
- **Light-Front Holography: $AdS_5 = LF (3+1)$**

$$z \leftrightarrow \zeta \text{ where } \zeta^2 = b_{\perp}^2 x(1-x)$$



- **Introduce Mass Scale κ while retaining the Conformal Invariance of the AdS Action (dAFF)**
- **Unique Dilaton in AdS_5 : $e^{+\kappa^2 z^2}$**
- **Unique color-confining LF Potential $U(\zeta^2) = \kappa^4 \zeta^2$**
- **Superconformal Algebra: Mass Degenerate 4-Plet:**

Meson $q\bar{q} \leftrightarrow$ Baryon $q[qq] \leftrightarrow$ Tetraquark $[qq][\bar{q}\bar{q}]$



$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$

Equal Slope in n and L

Quark separation
increases with L

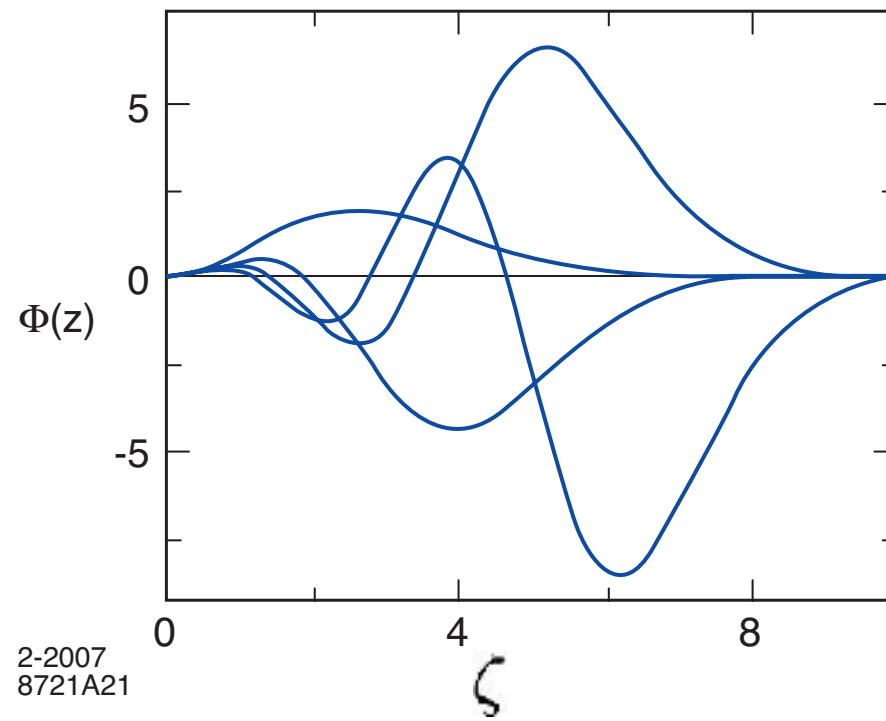
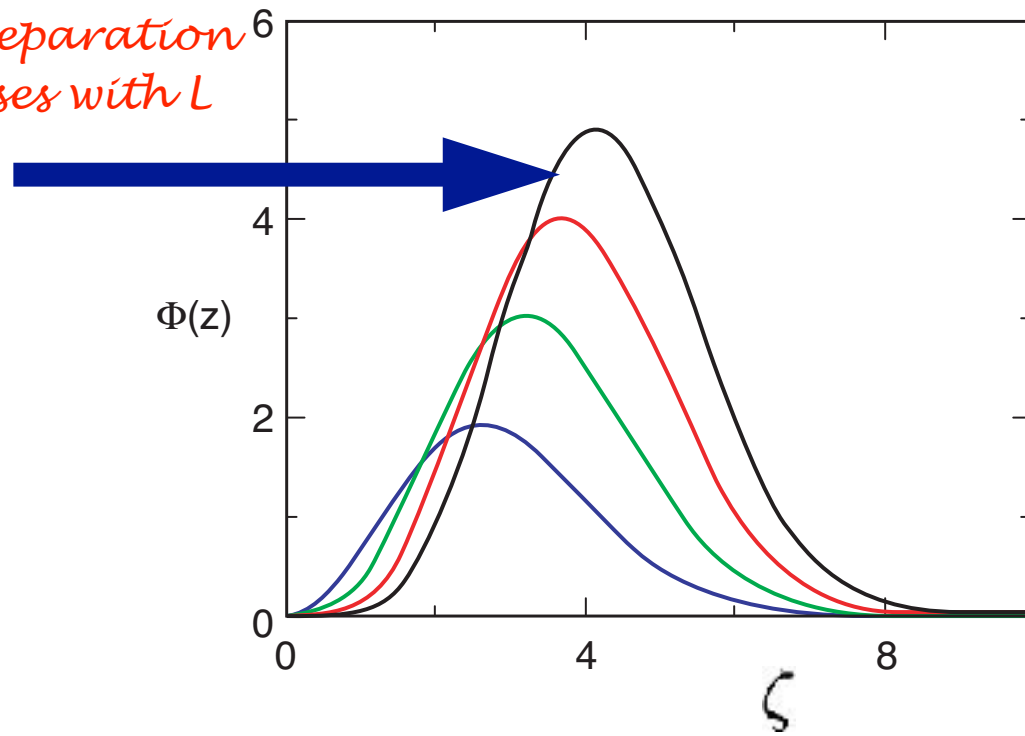
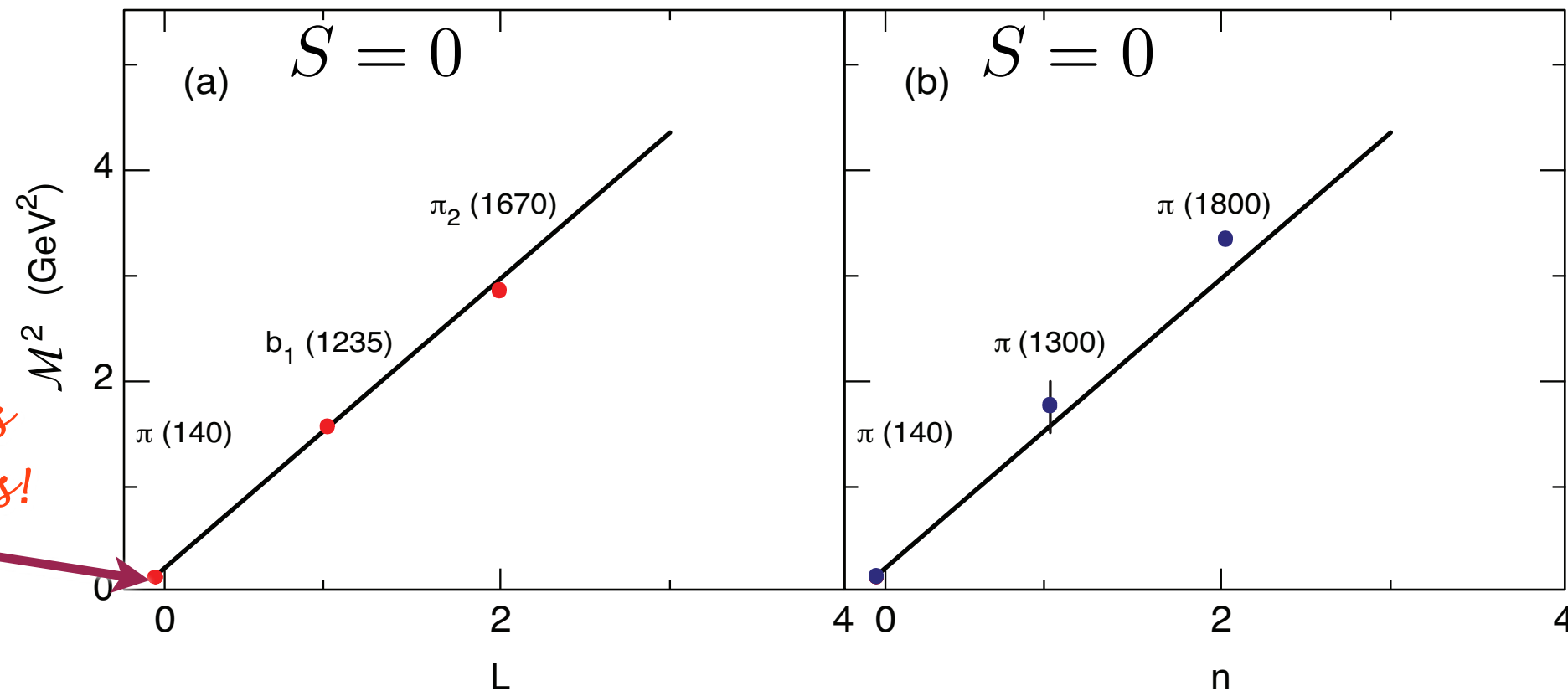


Fig: Orbital and radial AdS modes in the soft wall model for $\kappa = 0.6$ GeV .

Same slope in n and L !

*Soft Wall
Model*



Pion has
zero mass!

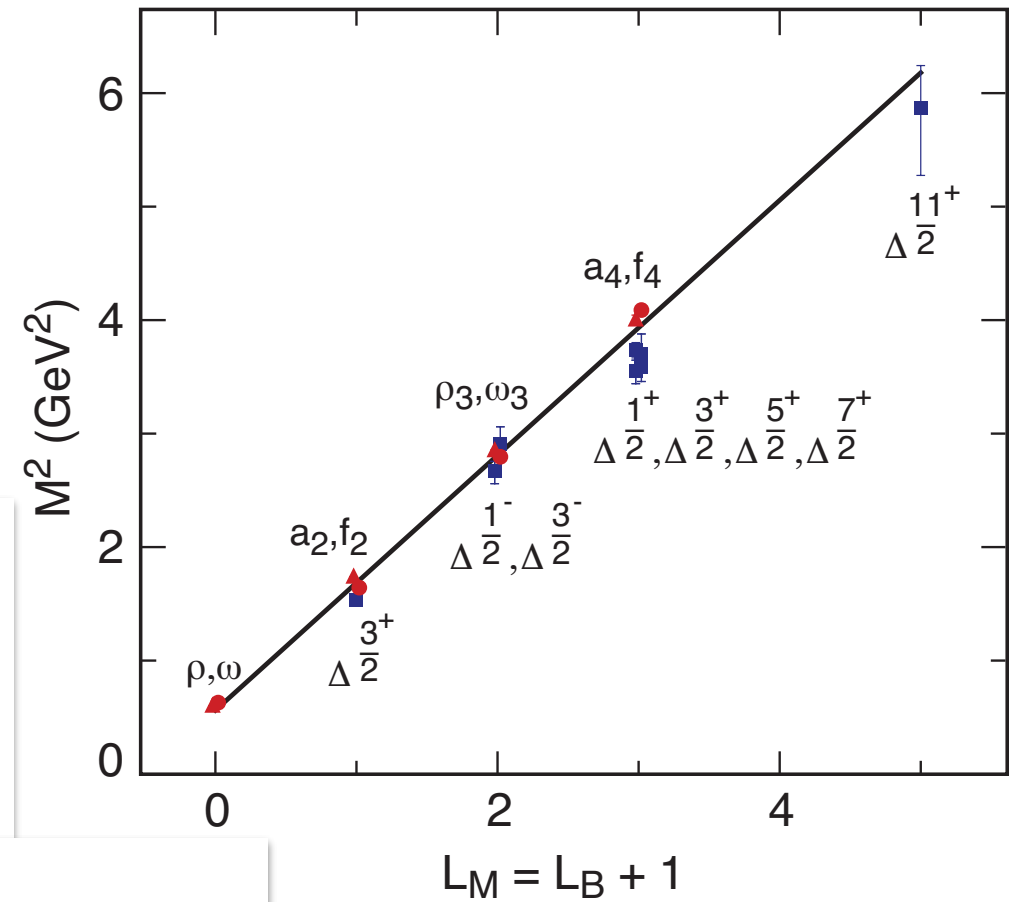
$$m_q = 0$$

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

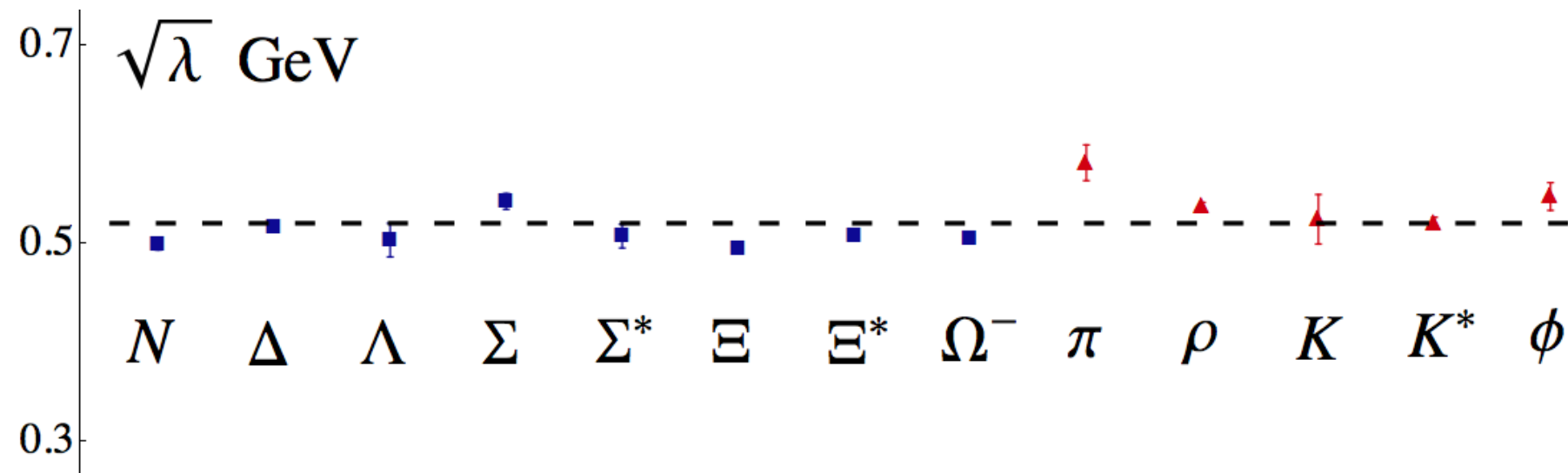
Universal Regge Slope in L and n

•

$$\kappa = \sqrt{\lambda} = 0.523 \pm 0.024$$



• How universal is the semiclassical approximation based on superconformal LFHQCD ?



Best fit for hadronic scale $\sqrt{\lambda}$ from different light hadron sectors including radial and orbital excitations

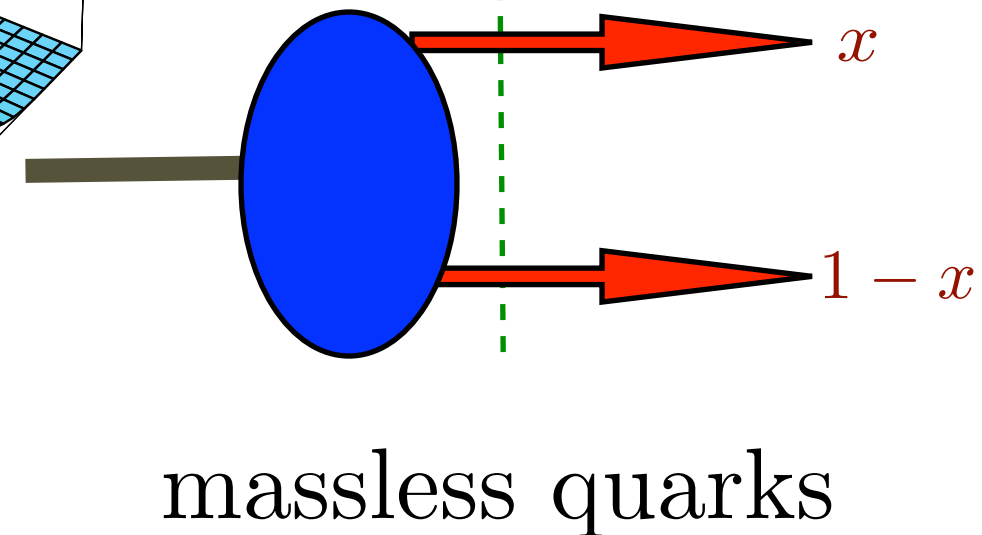
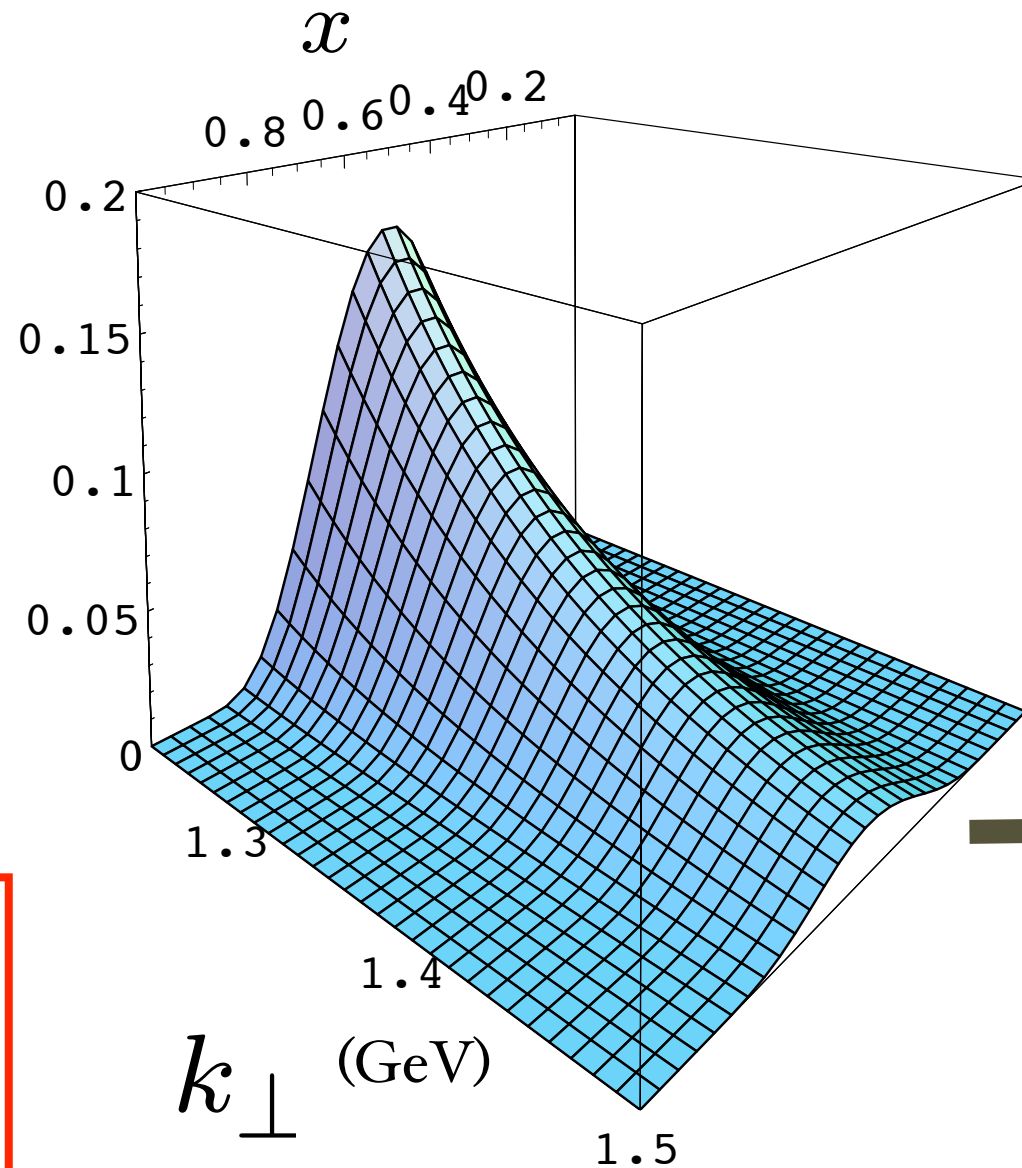
Prediction from AdS/QCD: Meson LFWF

$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

**de Teramond,
Cao, sjb**

**“Soft Wall”
model**

$$\psi_M(x, k_\perp^2)$$



Note coupling

$$k_\perp^2, x$$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

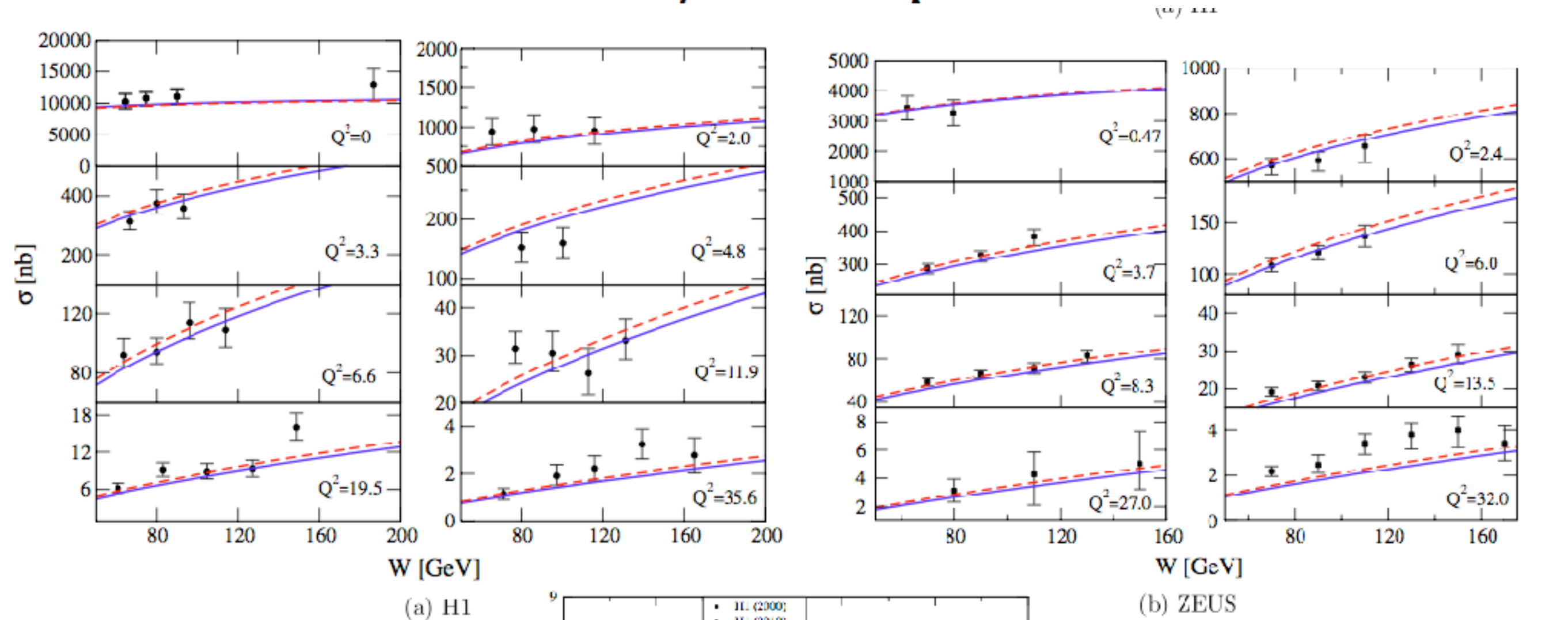
$$\phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}$$

$$f_\pi = \sqrt{P_{q\bar{q}}} \frac{\sqrt{3}}{8} \kappa = 92.4 \text{ MeV}$$

Same as DSE! **C. D. Roberts et al.**

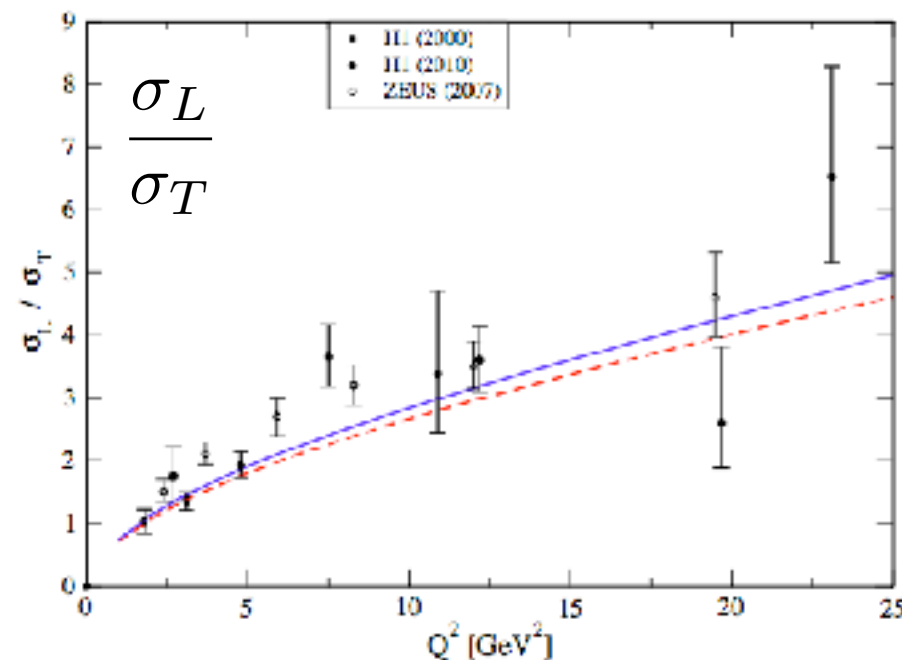
Provides Connection of Confinement to Hadron Structure

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction



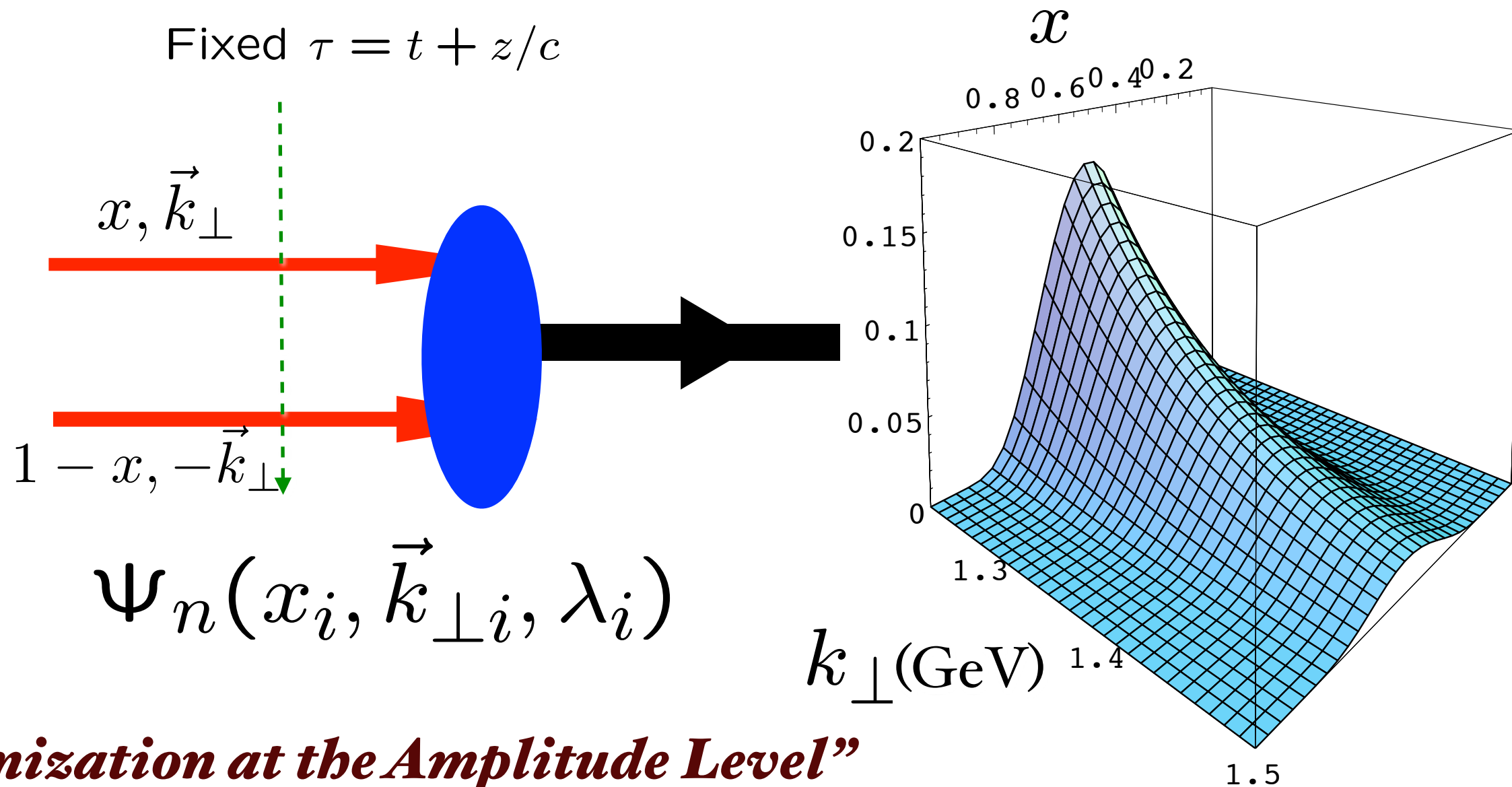
**J. R. Forshaw,
R. Sandapen**

$$\gamma^* p \rightarrow \rho^0 p'$$



$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

- Light Front Wavefunctions:* $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$
off-shell in P^- and invariant mass $\mathcal{M}_{q\bar{q}}^2$



“Hadronization at the Amplitude Level”

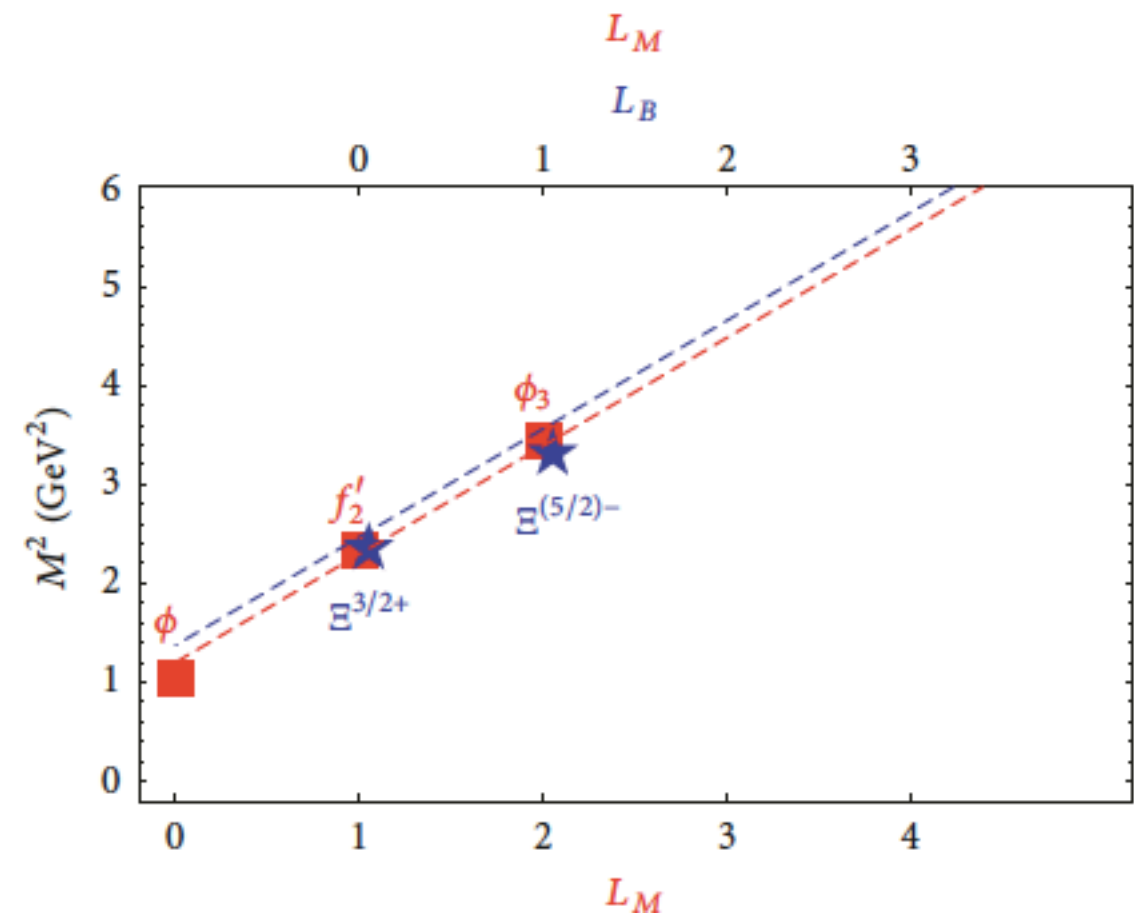
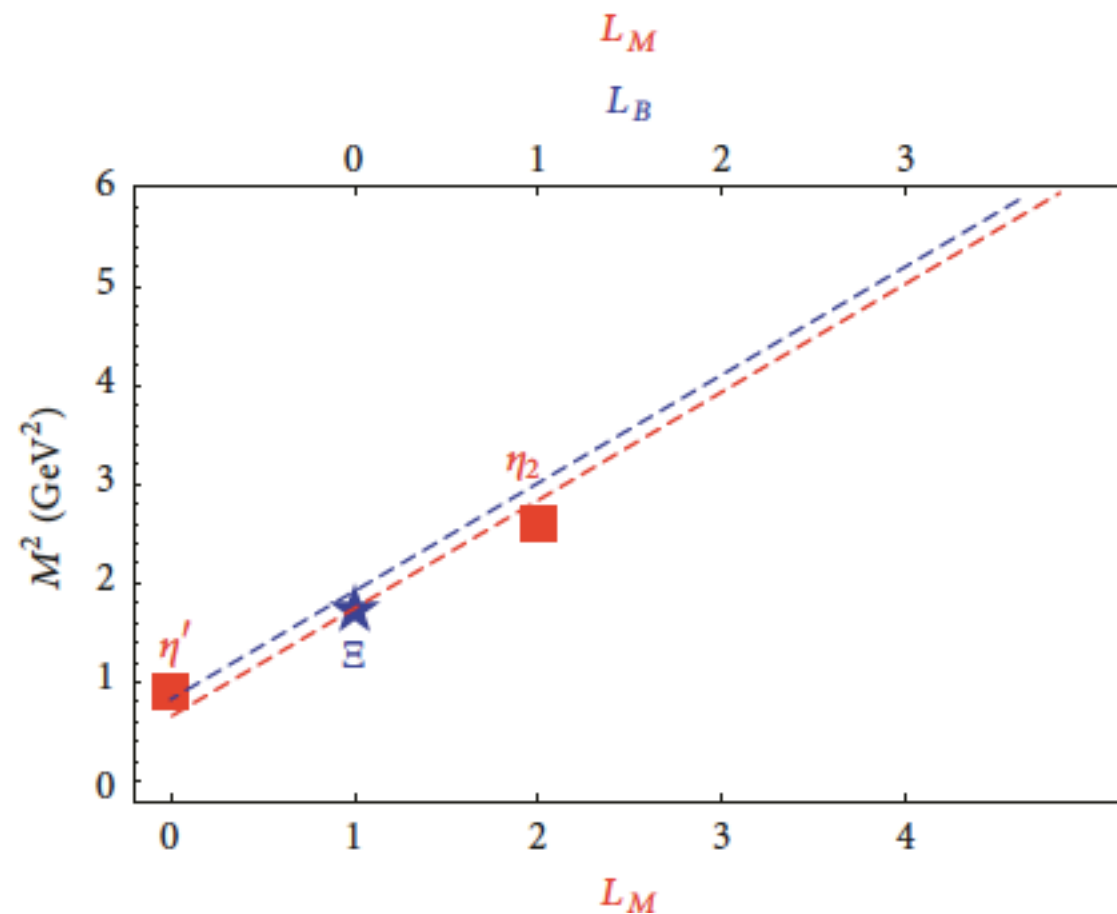
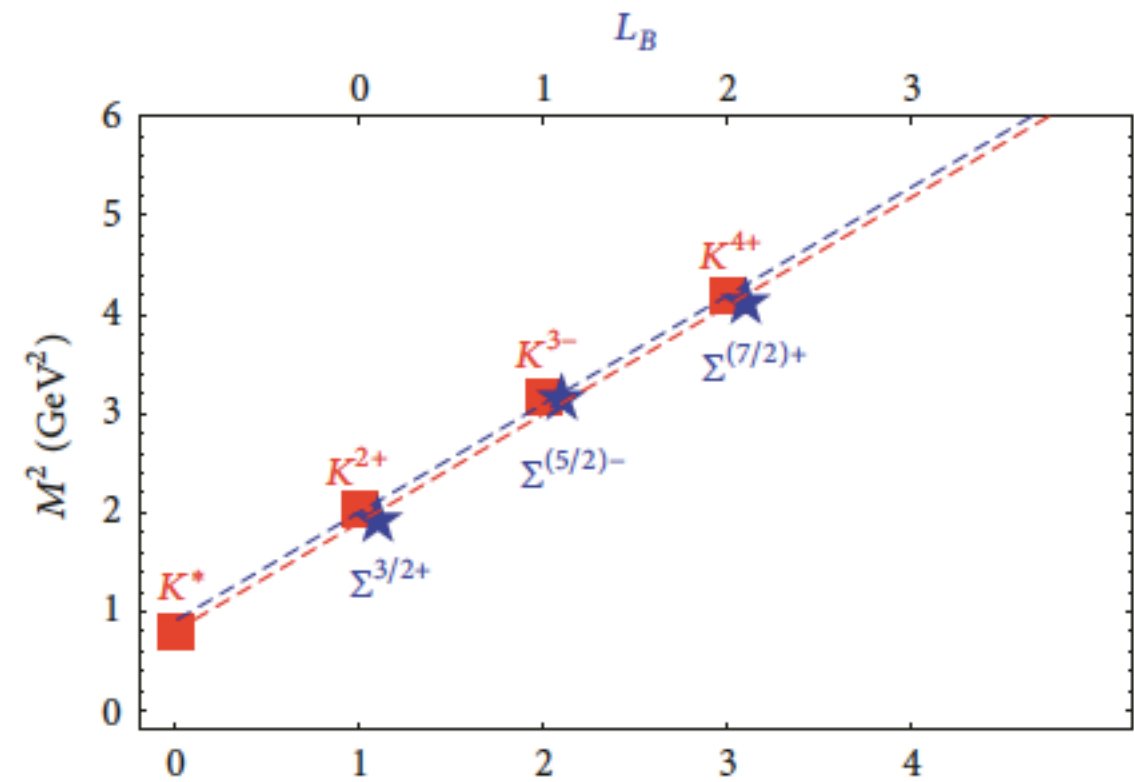
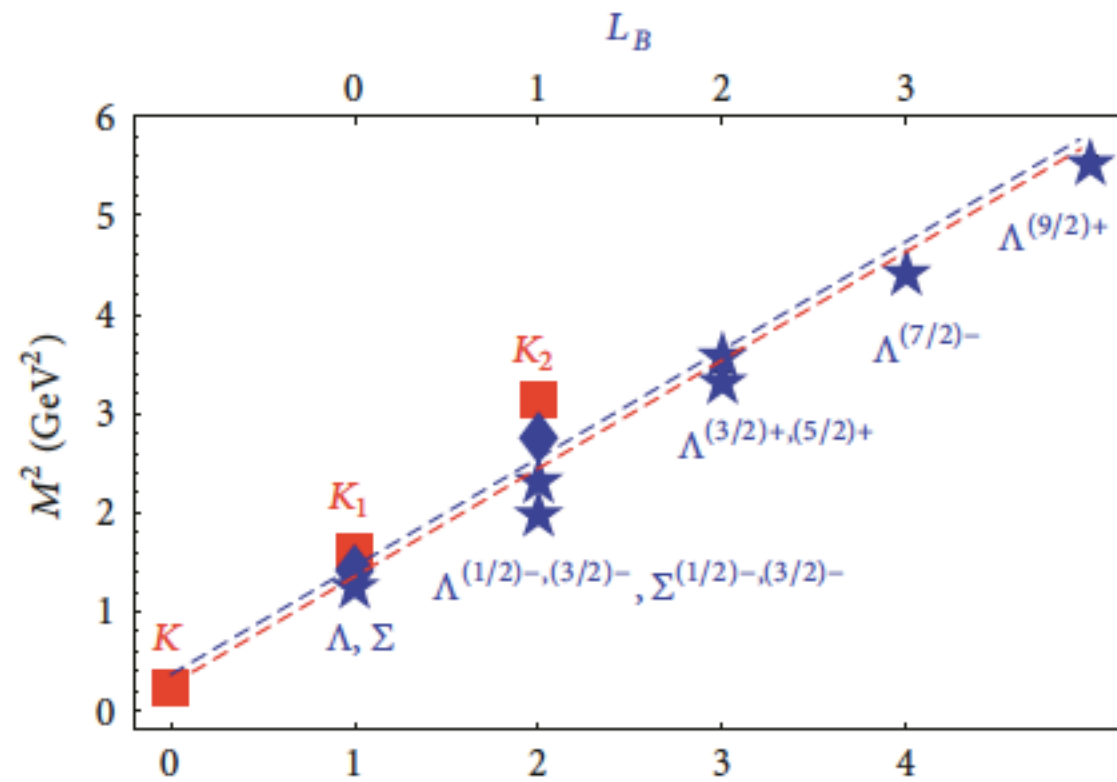
Boost-invariant LFWF connects confined quarks and gluons to hadrons

Light-Front Holography: First Approximation to QCD

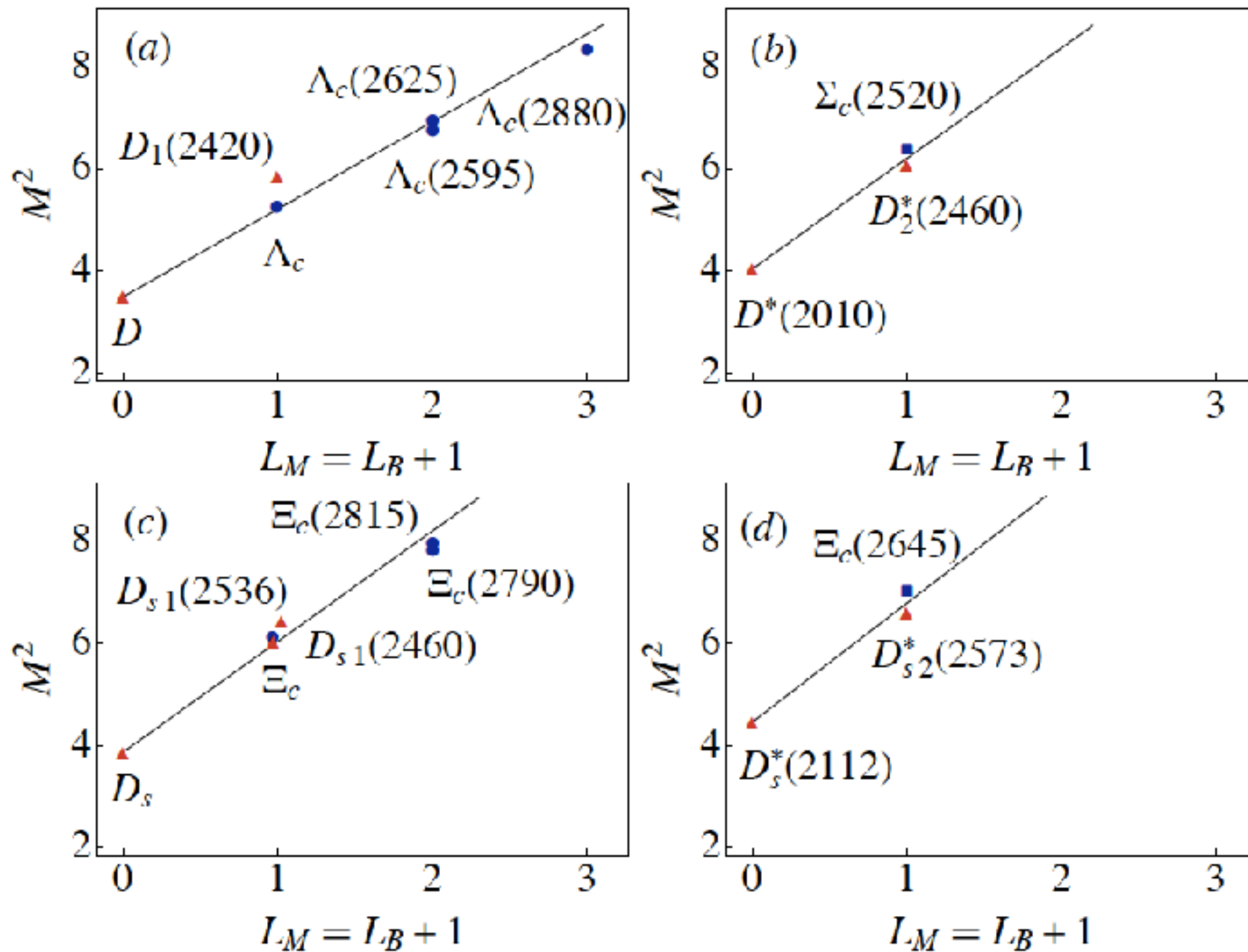
- **Color Confinement, Analytic form of confinement potential**
de Téramond, Dosch, Lorcé, sjb
- **Retains underlying conformal properties of QCD despite mass scale (DeAlfaro-Fubini-Furlan Principle)**
- **Massless quark-antiquark pion bound state in chiral limit, GMOR**
- **QCD coupling at all scales**
- **Connection of perturbative and nonperturbative mass scales**
- **Poincaré Invariant**
- **Hadron Spectroscopy-Regge Trajectories with universal slopes in n , L**
- **Supersymmetric 4-Plet: Meson-Baryon-Tetraquark Symmetry**
- **Light-Front Wavefunctions**
- **Form Factors, Structure Functions, Hadronic Observables**
- **OPE: Constituent Counting Rules**
- **Hadronization at the Amplitude Level: Many Phenomenological Tests**
- **Systematically improvable: Basis LF Quantization (BLFQ)**

*Supersymmetric Features of Hadron Physics
from Superconformal Algebra
and Light-Front Holography*

Supersymmetry across the light and heavy-light spectrum



Supersymmetry across the light and heavy-light spectrum

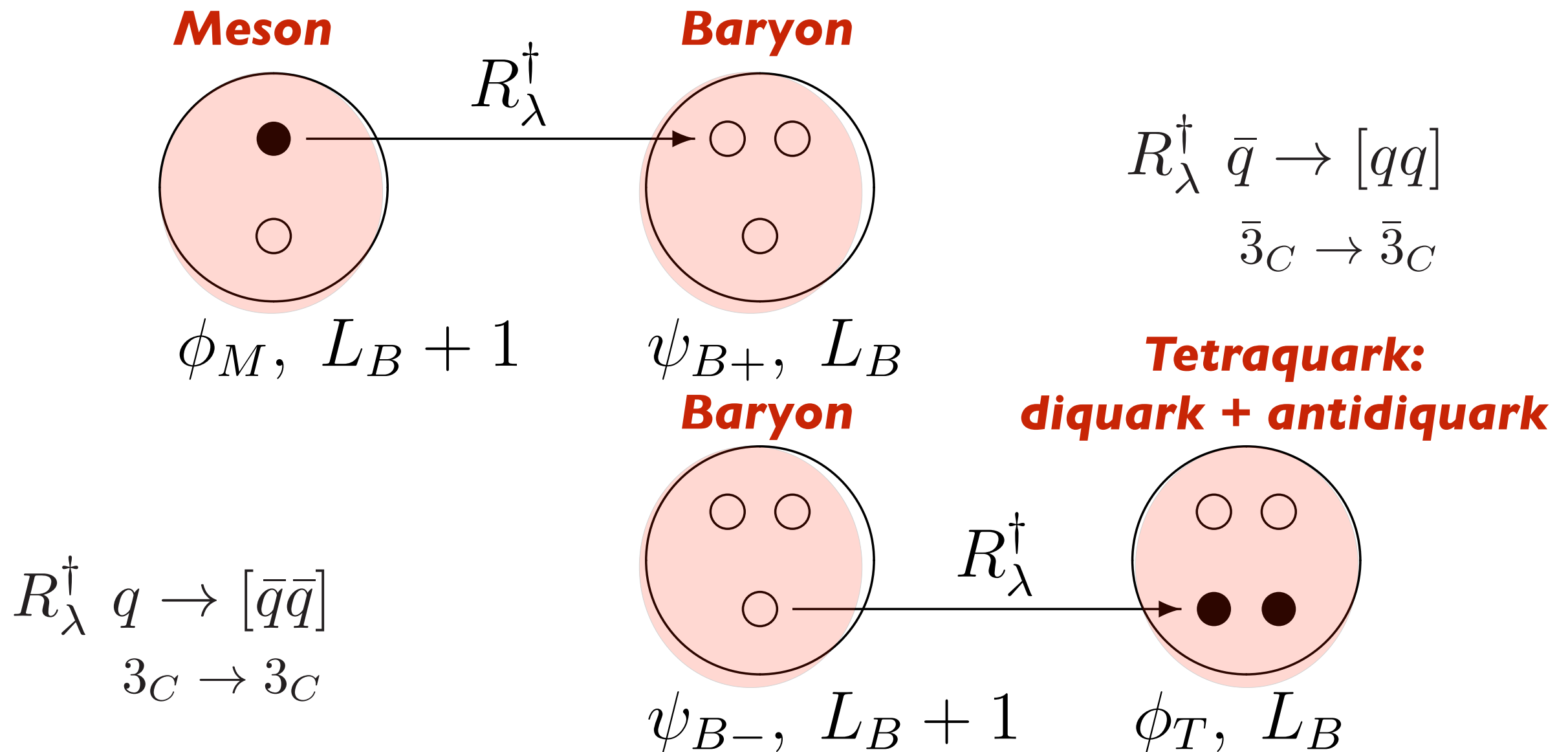


Heavy charm quark mass does not break supersymmetry

Superconformal Algebra

Four-Plet Representations

Bosons, Fermions with Equal Mass!



Proton: $|u[ud]\rangle$ Quark + Scalar Diquark
 Equal Weight: $L=0, L=1$

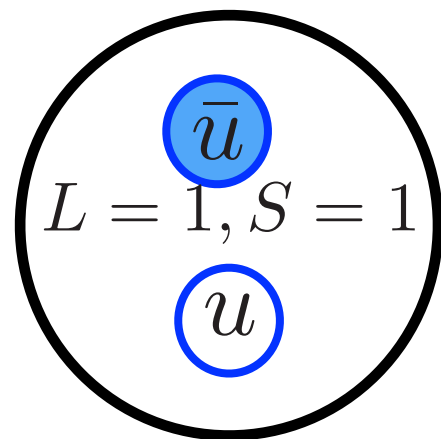
Superconformal Algebra 4-Plet

$$R_{\lambda}^{\dagger} \quad \bar{q} \rightarrow (qq) \quad S = 1$$

$$\bar{3}_C \rightarrow \bar{3}_C$$

Vector ()+ Scalar [] Diquarks

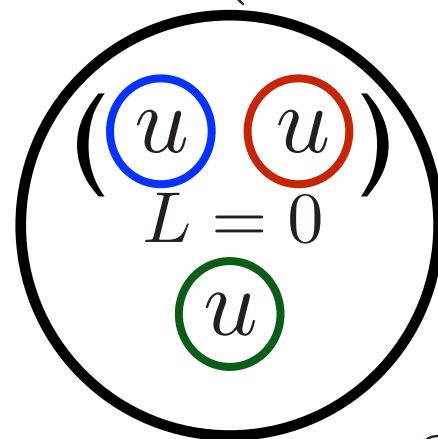
$f_2(1270)$



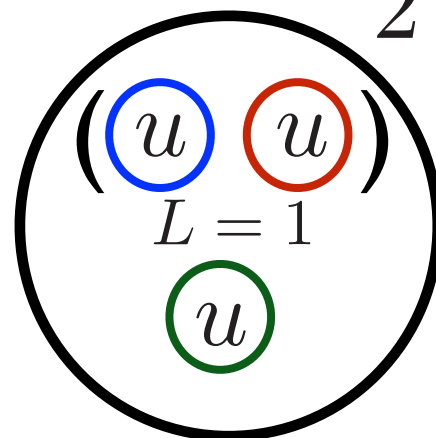
$$J^{PC} = 2^{++}$$

Meson

$\Delta^+(1232)$



$$J^P = \frac{3}{2}^+$$

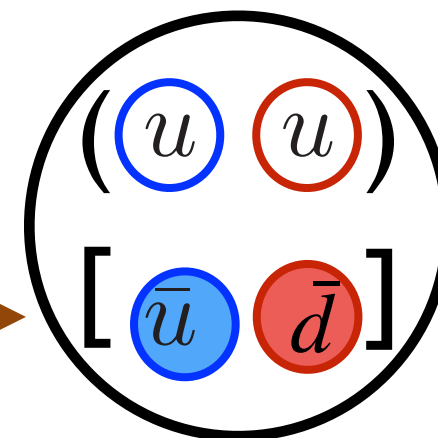


Baryon

Tetraquark

$$J^{PC} = 1^{++}$$

$a_1(1260)$



$$S = 0$$

$$L = 0$$

$$R_{\lambda}^{\dagger} \quad q \rightarrow [\bar{q}\bar{q}]$$

$$3_C \rightarrow 3_C$$

Superconformal meson-baryon-tetraquark symmetry

H. G. Dosch, G. d-Te'ramond, sjb, PRD 91, 085016 (2015)

Upon the substitution in the superconformal equations

$$x \mapsto \zeta, \quad E \mapsto M^2,$$

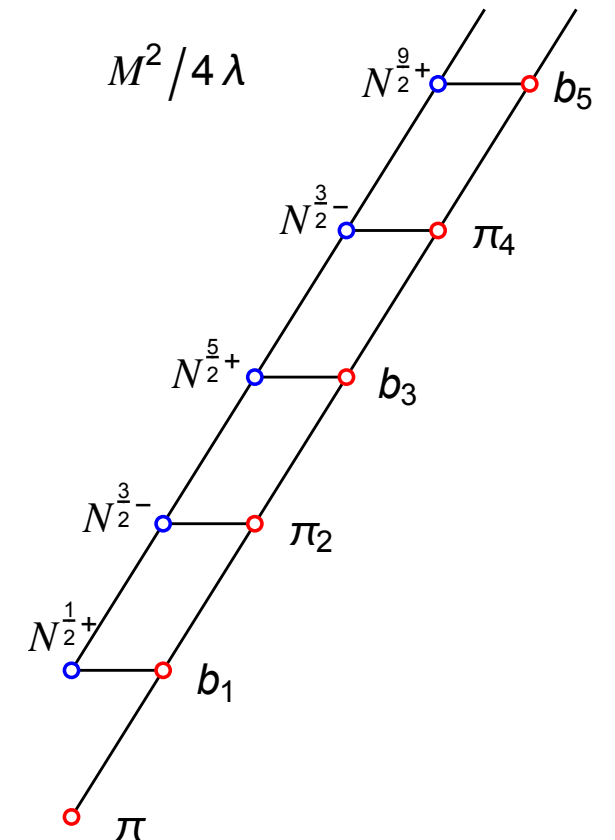
$$\lambda \mapsto \lambda_B = \lambda_M, \quad f \mapsto L_M - \frac{1}{2} = L_B + \frac{1}{2}$$

$$\phi_1 \mapsto \phi_M, \quad \phi_2 \mapsto \phi_B$$

we find the LF meson/baryon bound-state equations

$$\left(-\frac{d^2}{d\zeta^2} + \frac{4L_M^2 - 1}{4\zeta^2} + \lambda_M^2 \zeta^2 + 2\lambda_M(L_M - 1) \right) \phi_M = M^2 \phi_M$$

$$\left(-\frac{d^2}{d\zeta^2} + \frac{4L_B^2 - 1}{4\zeta^2} + \lambda_B^2 \zeta^2 + 2\lambda_B(L_B + 1) \right) \phi_B = M^2 \phi_B$$



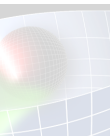
$$\Phi = \begin{pmatrix} \phi_M & \phi_B^- \\ \phi_B^+ & \phi_T \end{pmatrix}$$

Superconformal QM imposes the condition $\lambda = \lambda_M = \lambda_B$ (equality of Regge slopes) and the remarkable relation $L_M = L_B + 1$

L_M is the LF angular momentum between the quark and antiquark in the meson and L_B between the active quark and spectator diquark cluster in the baryon

Full hadron 4-plet: meson-baryon-tetraquark

G d-Te'ramond, H. G. Dosch and C. Lorce, PLB 759, 171 (2016)



Meson			Baryon			Tetraquark		
q -cont	$J^{P(C)}$	Name	q -cont	J^P	Name	q -cont	$J^{P(C)}$	Name
$\bar{q}q$	0^{-+}	$\pi(140)$	—	—	—	—	—	—
$\bar{q}q$	1^{+-}	$b_1(1235)$	$[ud]q$	$(1/2)^+$	$N(940)$	$[ud][\bar{u}\bar{d}]$	0^{++}	$f_0(980)$
$\bar{q}q$	2^{-+}	$\pi_2(1670)$	$[ud]q$	$(1/2)^-$	$N_{\frac{1}{2}}(1535)$	$[ud][\bar{u}\bar{d}]$	1^{-+}	$\pi_1(1400)$
				$(3/2)^-$	$N_{\frac{3}{2}}(1520)$			$\pi_1(1600)$
$\bar{q}q$	1^{--}	$\rho(770), \omega(780)$	—	—	—	—	—	—
$\bar{q}q$	2^{++}	$a_2(1320), f_2(1270)$	$[qq]q$	$(3/2)^+$	$\Delta(1232)$	$[qq][\bar{u}\bar{d}]$	1^{++}	$a_1(1260)$
$\bar{q}q$	3^{--}	$\rho_3(1690), \omega_3(1670)$	$[qq]q$	$(1/2)^-$	$\Delta_{\frac{1}{2}}(1620)$	$[qq][\bar{u}\bar{d}]$	2^{--}	$\rho_2(\sim 1700)?$
				$(3/2)^-$	$\Delta_{\frac{3}{2}}(1700)$			
$\bar{q}q$	4^{++}	$a_4(2040), f_4(2050)$	$[qq]q$	$(7/2)^+$	$\Delta_{\frac{7}{2}}(1950)$	$[qq][\bar{u}\bar{d}]$	3^{++}	$a_3(\sim 2070)?$
$\bar{q}s$	$0^{-(+)}$	$\bar{K}(495)$	—	—	—	—	—	—
$\bar{q}s$	$1^{+(-)}$	$\bar{K}_1(1270)$	$[ud]s$	$(1/2)^+$	$\Lambda(1115)$	$[ud][\bar{s}\bar{q}]$	$0^{+(+)}$	$K_0^*(1430)$
$\bar{q}s$	$2^{-(+)}$	$K_2(1770)$	$[ud]s$	$(1/2)^-$	$\Lambda(1405)$	$[ud][\bar{s}\bar{q}]$	$1^{-(+)}$	$K_1^*(\sim 1700)?$
				$(3/2)^-$	$\Lambda(1520)$			
$\bar{s}q$	$0^{-(+)}$	$K(495)$	—	—	—	—	—	—
$\bar{s}q$	$1^{+(-)}$	$K_1(1270)$	$[sq]q$	$(1/2)^+$	$\Sigma(1190)$	$[sq][\bar{s}\bar{q}]$	0^{++}	$a_0(980)$ $f_0(980)$
$\bar{s}q$	$1^{-(-)}$	$K^*(890)$	—	—	—	—	—	—
$\bar{s}q$	$2^{+(+)}$	$K_2^*(1430)$	$[sq]q$	$(3/2)^+$	$\Sigma(1385)$	$[sq][\bar{q}\bar{q}]$	$1^{+(+)}$	$K_1(1400)$
$\bar{s}q$	$3^{-(-)}$	$K_3^*(1780)$	$[sq]q$	$(3/2)^-$	$\Sigma(1670)$	$[sq][\bar{q}\bar{q}]$	$2^{-(-)}$	$K_2(\sim 1700)?$
$\bar{s}q$	$4^{+(+)}$	$K_4^*(2045)$	$[sq]q$	$(7/2)^+$	$\Sigma(2030)$	$[sq][\bar{q}\bar{q}]$	$3^{+(+)}$	$K_3(\sim 2070)?$
$\bar{s}s$	0^{-+}	$\eta(550)$	—	—	—	—	—	—
$\bar{s}s$	1^{+-}	$h_1(1170)$	$[sq]s$	$(1/2)^+$	$\Xi(1320)$	$[sq][\bar{s}\bar{q}]$	0^{++}	$f_0(1370)$ $a_0(1450)$
$\bar{s}s$	2^{-+}	$\eta_2(1645)$	$[sq]s$	$(?)^?$	$\Xi(1690)$	$[sq][\bar{s}\bar{q}]$	1^{-+}	$\Phi'(1750)?$
$\bar{s}s$	1^{--}	$\Phi(1020)$	—	—	—	—	—	—
$\bar{s}s$	2^{++}	$f_2'(1525)$	$[sq]s$	$(3/2)^+$	$\Xi^*(1530)$	$[sq][\bar{s}\bar{q}]$	1^{++}	$f_1(1420)$
$\bar{s}s$	3^{--}	$\Phi_3(1850)$	$[sq]s$	$(3/2)^-$	$\Xi(1820)$	$[sq][\bar{s}\bar{q}]$	2^{--}	$\Phi_2(\sim 1800)?$
$\bar{s}s$	2^{++}	$f_2(1950)$	$[ss]s$	$(3/2)^+$	$\Omega(1672)$	$[ss][\bar{s}\bar{q}]$	$1^{+(+)}$	$K_1(\sim 1700)?$

Meson

Baryon

Tetraquark

New Organization of the Hadron Spectrum

M. Nielsen,
sjb

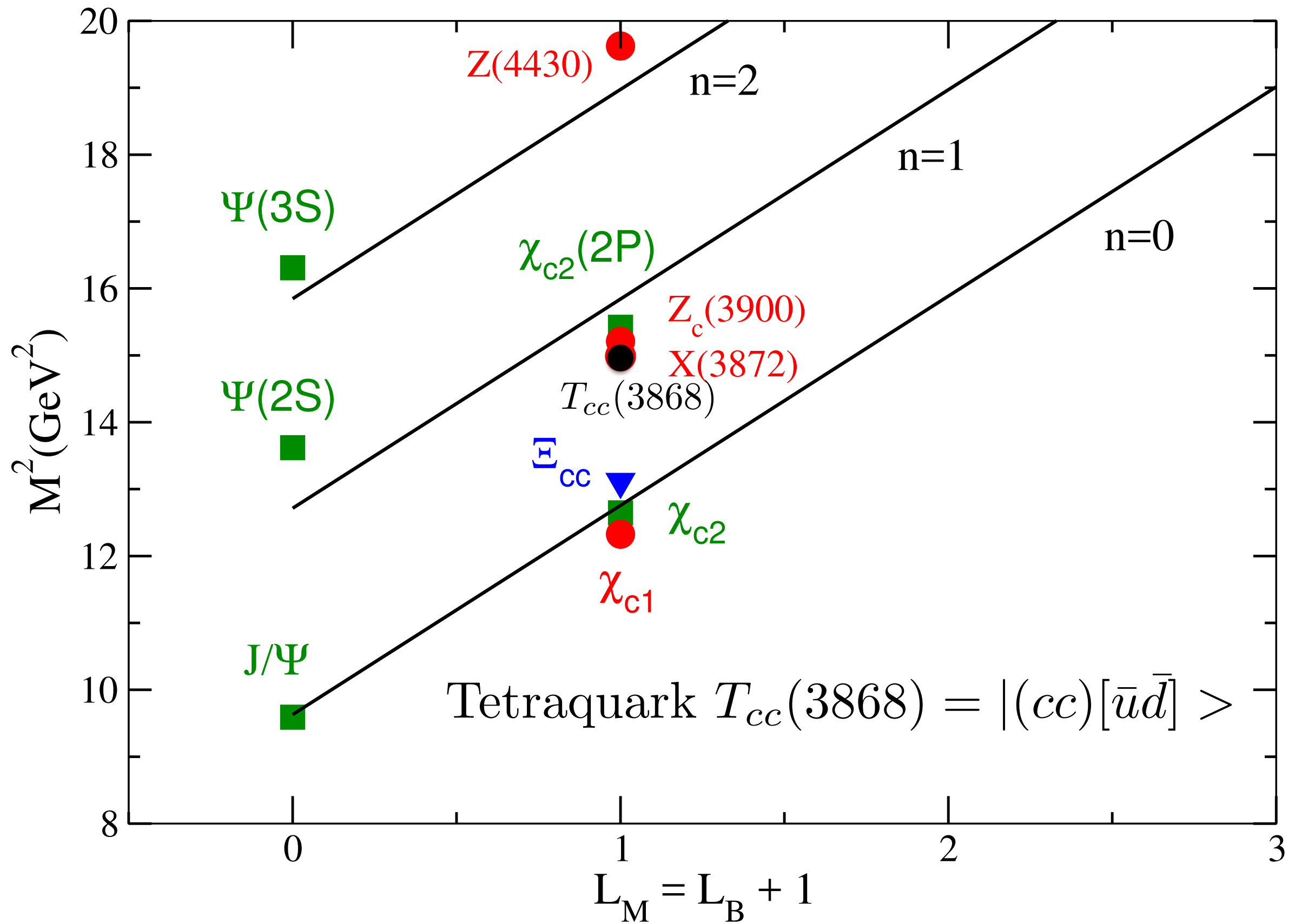
Superpartners for states with one c quark

Meson			Baryon			Tetraquark		
q -cont	$J^{P(C)}$	Name	q -cont	J^P	Name	q -cont	$J^{P(C)}$	Name
$\bar{q}c$	0^-	$D(1870)$	—	—	—	—	—	—
$\bar{q}c$	1^+	$D_1(2420)$	$[ud]c$	$(1/2)^+$	$\Lambda_c(2290)$	$[ud][\bar{c}\bar{q}]$	0^+	$\bar{D}_0^*(2400)$
$\bar{q}c$	2^-	$D_J(2600)$	$[ud]c$	$(3/2)^-$	$\Lambda_c(2625)$	$[ud][\bar{c}\bar{q}]$	1^-	—
$\bar{c}q$	0^-	$\bar{D}(1870)$	—	—	—	—	—	—
$\bar{c}q$	1^+	$\bar{D}_1(2420)$	$[cq]q$	$(1/2)^+$	$\Sigma_c(2455)$	$[cq][\bar{u}\bar{d}]$	0^+	$D_0^*(2400)$
$\bar{q}c$	1^-	$D^*(2010)$	—	—	—	—	—	—
$\bar{q}c$	2^+	$D_2^*(2460)$	$(qq)c$	$(3/2)^+$	$\Sigma_c^*(2520)$	$(qq)[\bar{c}\bar{q}]$	1^+	$D(2550)$
$\bar{q}c$	3^-	$D_3^*(2750)$	$(qq)c$	$(3/2)^-$	$\Sigma_c(2800)$	$(qq)[\bar{c}\bar{q}]$	—	—
$\bar{s}c$	0^-	$D_s(1968)$	—	—	—	—	—	—
$\bar{s}c$	1^+	$D_{s1}(2460)$	$[qs]c$	$(1/2)^+$	$\Xi_c(2470)$	$[qs][\bar{c}\bar{q}]$	0^+	$\bar{D}_{s0}^*(2317)$
$\bar{s}c$	2^-	$D_{s2}(\sim 2860)?$	$[qs]c$	$(3/2)^-$	$\Xi_c(2815)$	$[sq][\bar{c}\bar{q}]$	1^-	—
$\bar{s}c$	1^-	$D_s^*(2110)$	—	—	—	—	—	—
$\bar{s}c$	2^+	$D_{s2}^*(2573)$	$(sq)c$	$(3/2)^+$	$\Xi_c^*(2645)$	$(sq)[\bar{c}\bar{q}]$	1^+	$D_{s1}(2536)$
$\bar{c}s$	1^+	$D_{s1}(\sim 2700)?$	$[cs]s$	$(1/2)^+$	$\Omega_c(2695)$	$[cs][\bar{s}\bar{q}]$	0^+	??
$\bar{s}c$	2^+	$D_{s2}^*(\sim 2750)?$	$(ss)c$	$(3/2)^+$	$\Omega_c(2770)$	$(ss)[\bar{c}\bar{s}]$	1^+	??

M. Nielsen, sjb

predictions

beautiful agreement!



Mesons : *GreenSquare*, Baryons(*BlueTriangle*), Tetraquarks(*RedCircle*)

Connection to the Linear Instant-Form Heavy Quark Potential

Harmonic Oscillator $U(\zeta) = \kappa^4 \zeta^2$ LF Potential for relativistic light quarks



Linear instant nonrelativistic form $V(r) = Cr$ for heavy quarks

A.P. Trawinski, S.D. Glazek, H. D. Dosch, G. de Teramond, sjb

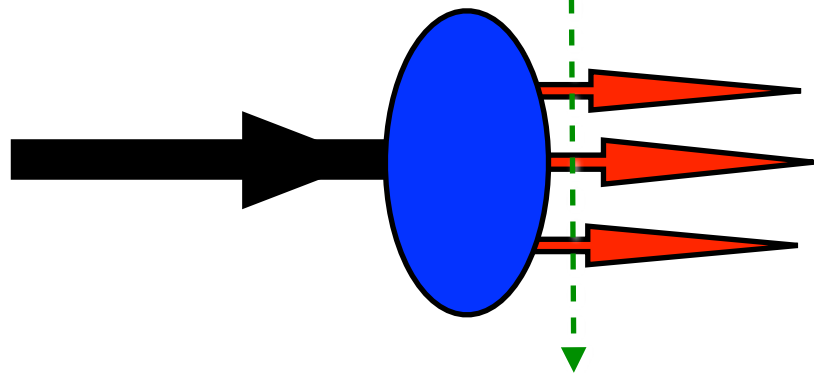
Bound States in Relativistic Quantum Field Theory:

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$

Fixed $\tau = t + z/c$

Boost invariant, Lorentz frame independent, Causal



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Invariant under boosts. Independent of P^μ

$$H_{LF}^{QCD} |\psi\rangle = M^2 |\psi\rangle$$

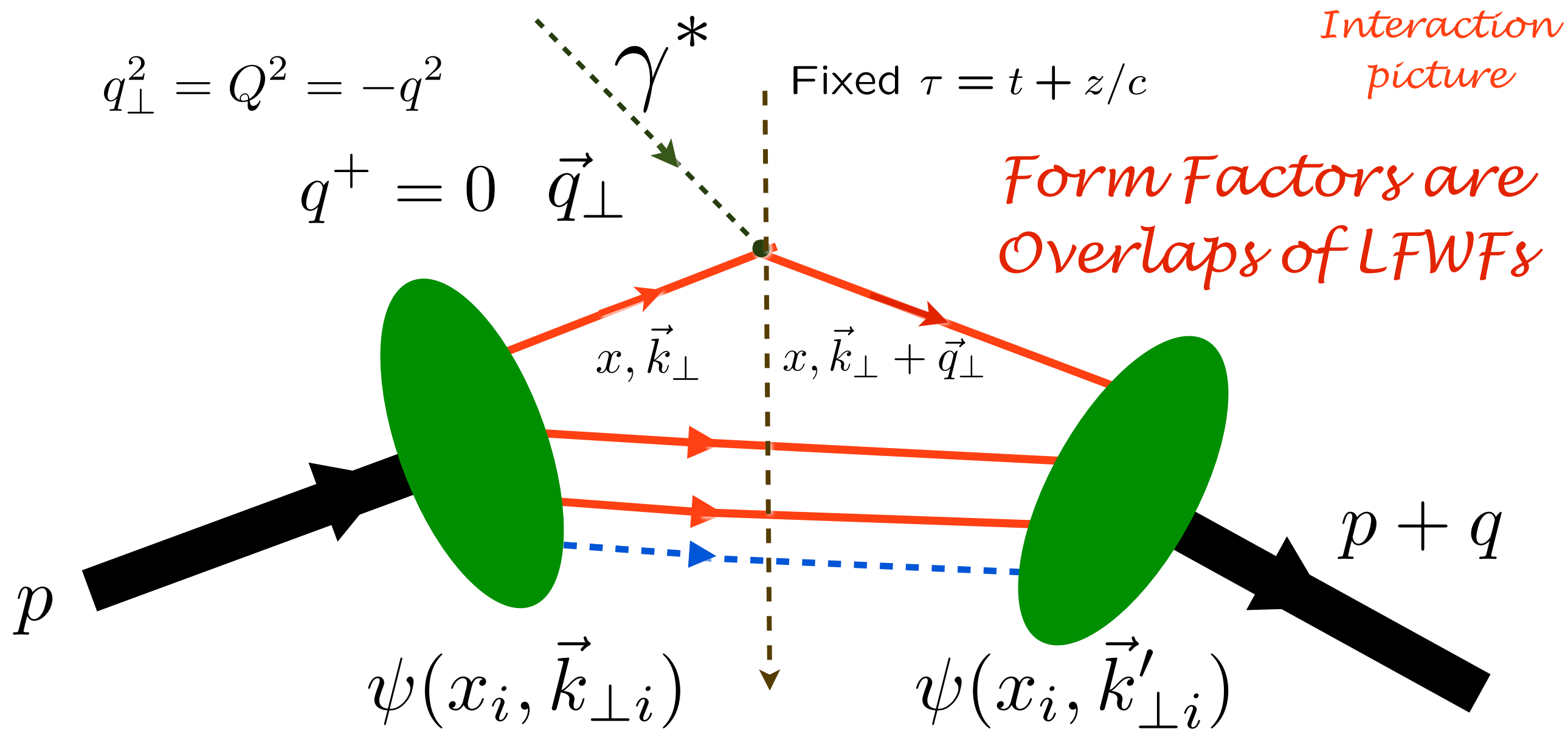
Direct connection to QCD Lagrangian

LF Wavefunction: off-shell in invariant mass

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

$$\langle p+q | j^+(0) | p \rangle = 2p^+ F(q^2)$$

Front Form

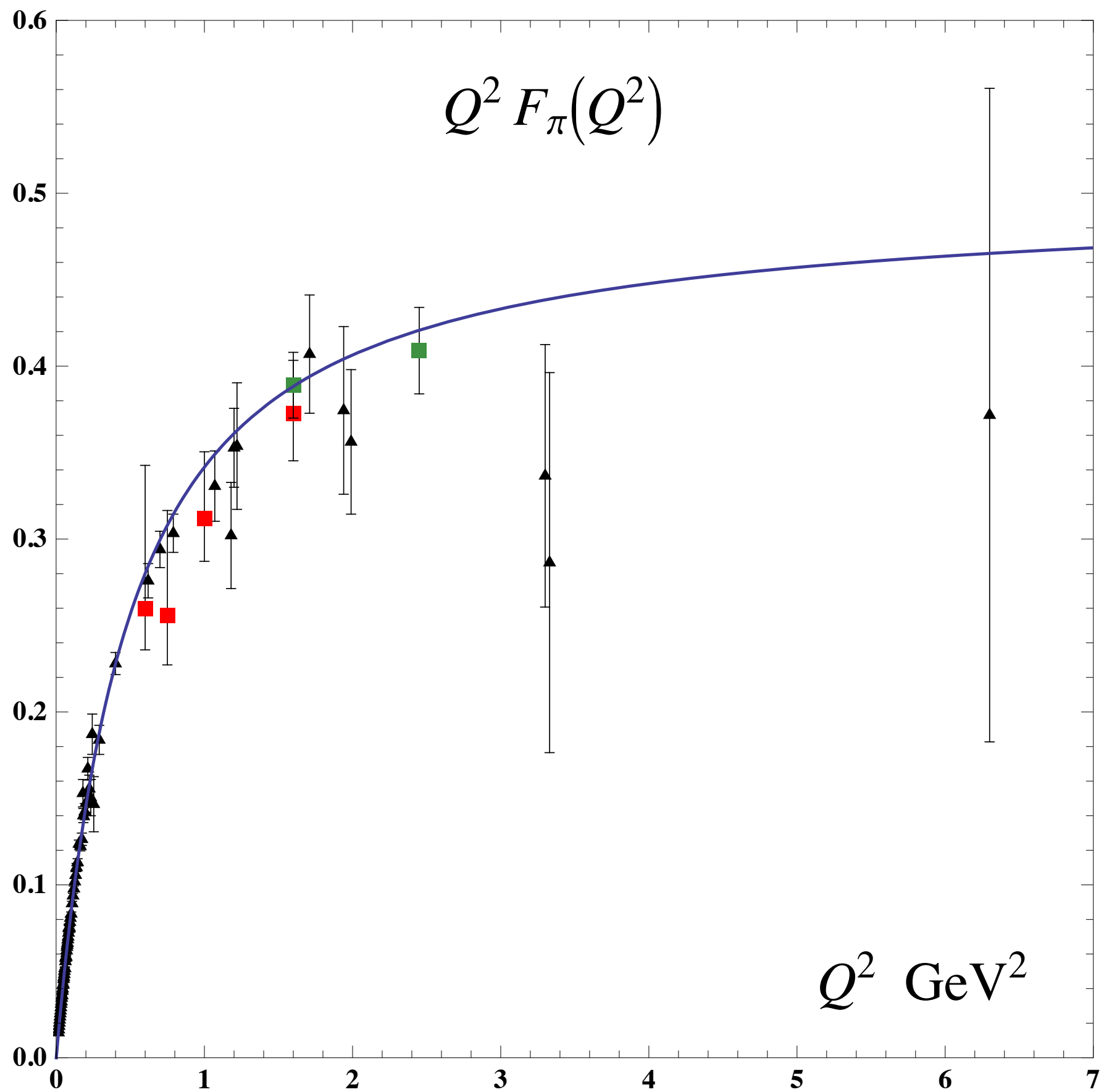


Drell & Yan, West
Exact LF formula!

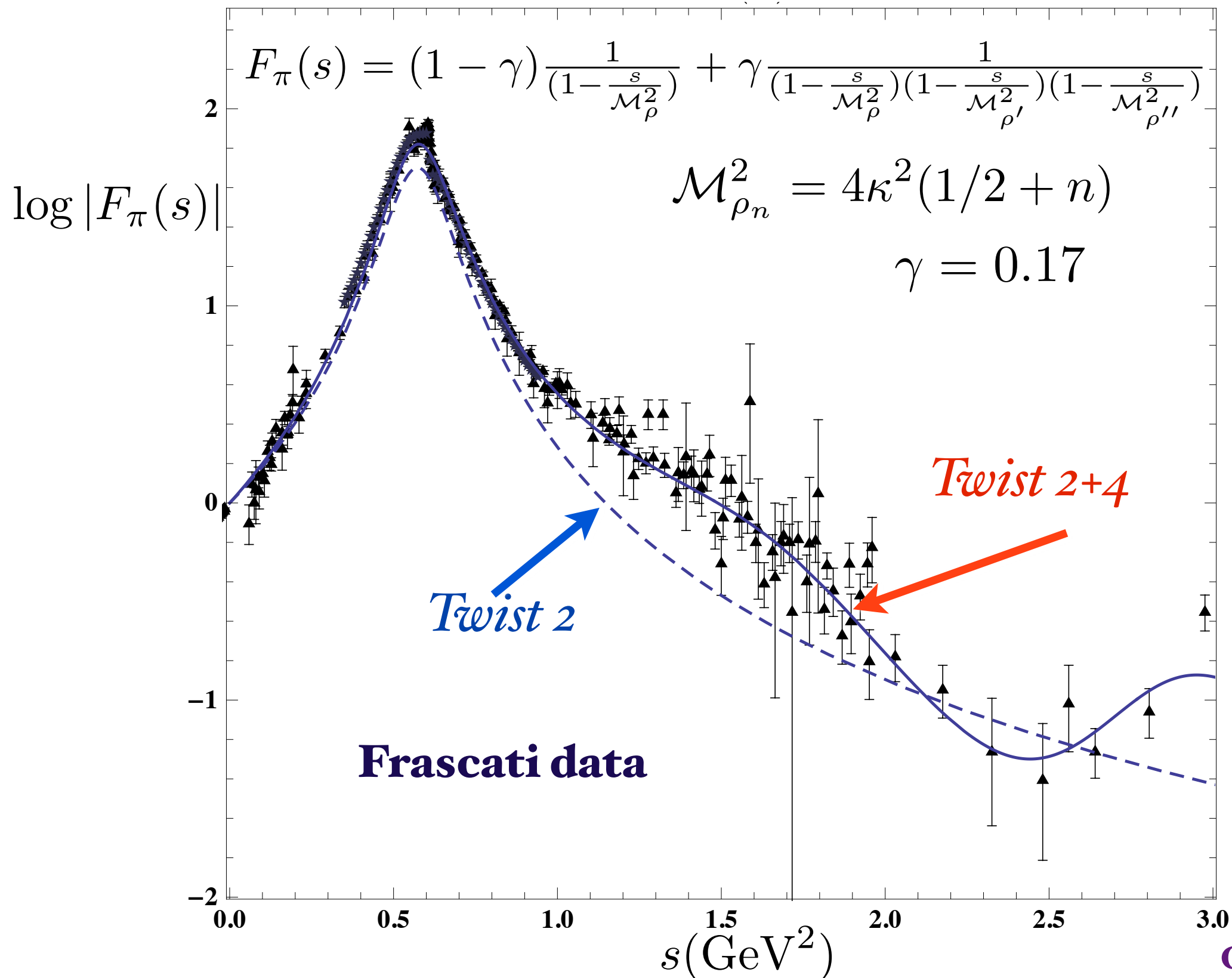
struck $\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i) \vec{q}_{\perp}$
spectators $\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i \vec{q}_{\perp}$

Drell, sjb

Transverse size $\propto \frac{1}{Q}$



Timelike Pion Form Factor from AdS/QCD and Light-Front Holography



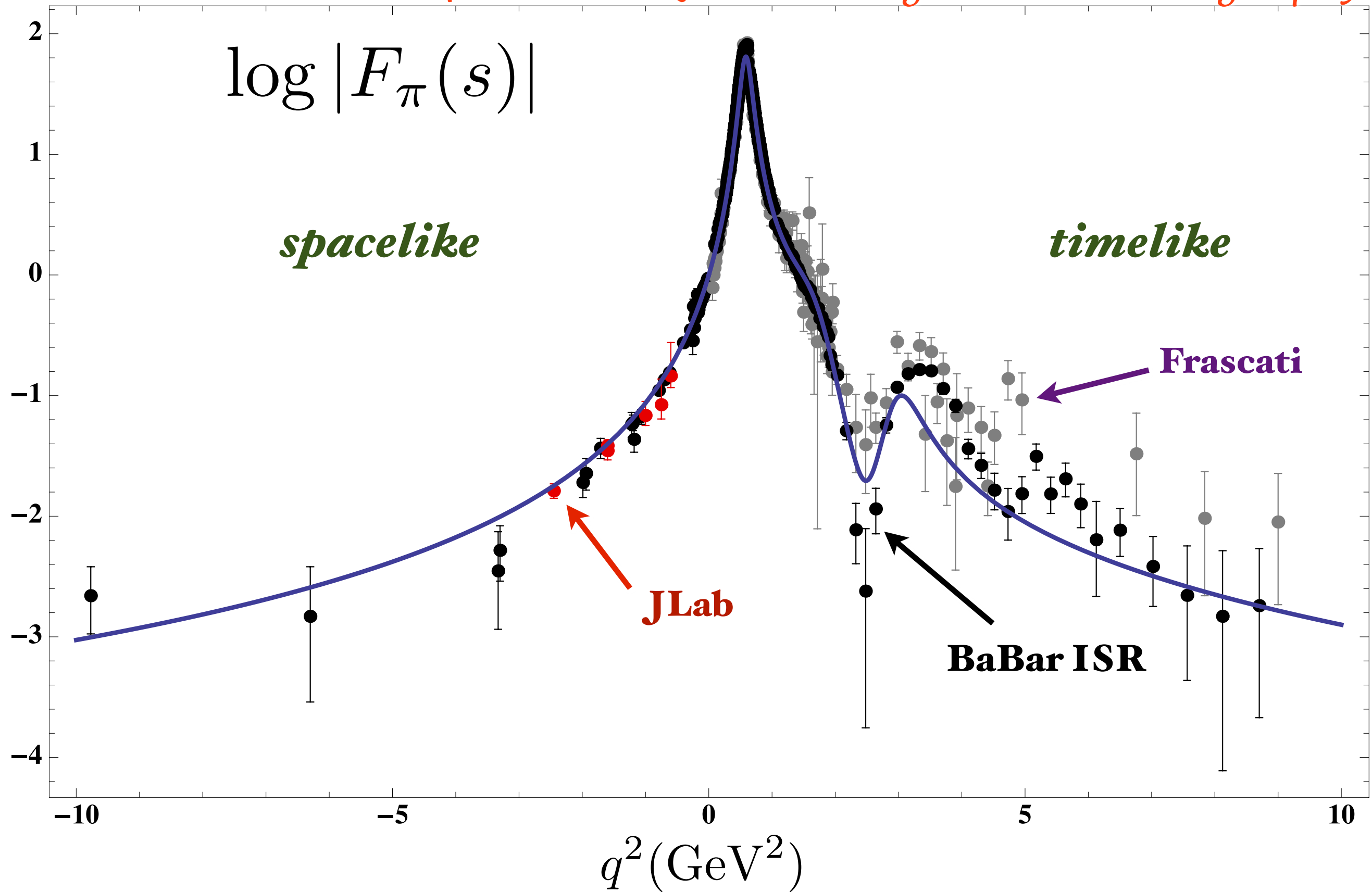
**Prescription for
Timelike poles :**

$$\frac{1}{s - M^2 + i\sqrt{s}\Gamma}$$

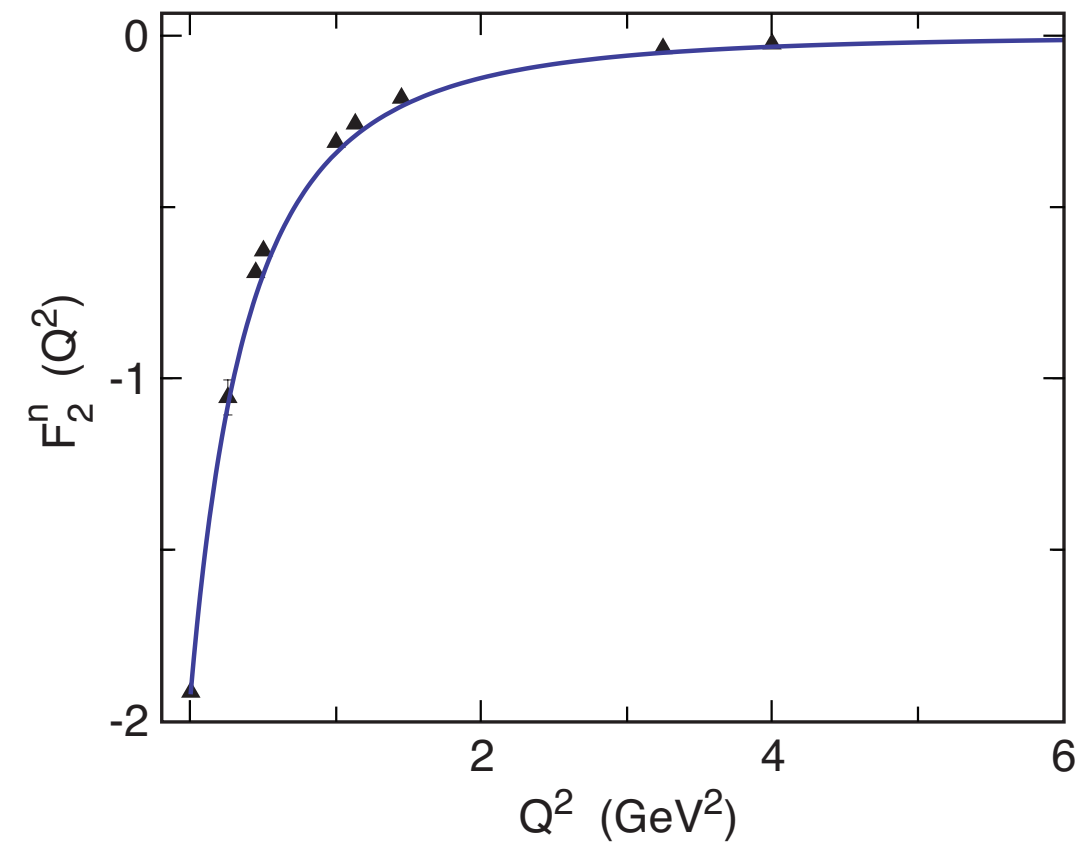
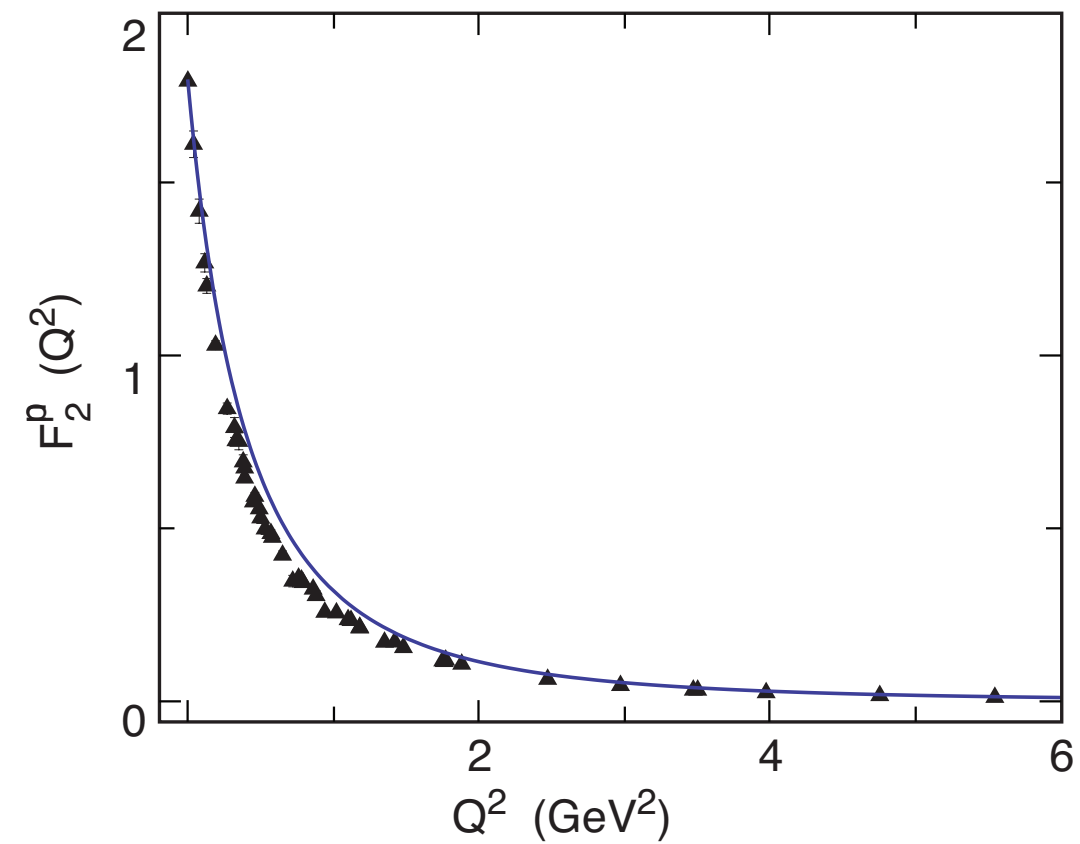
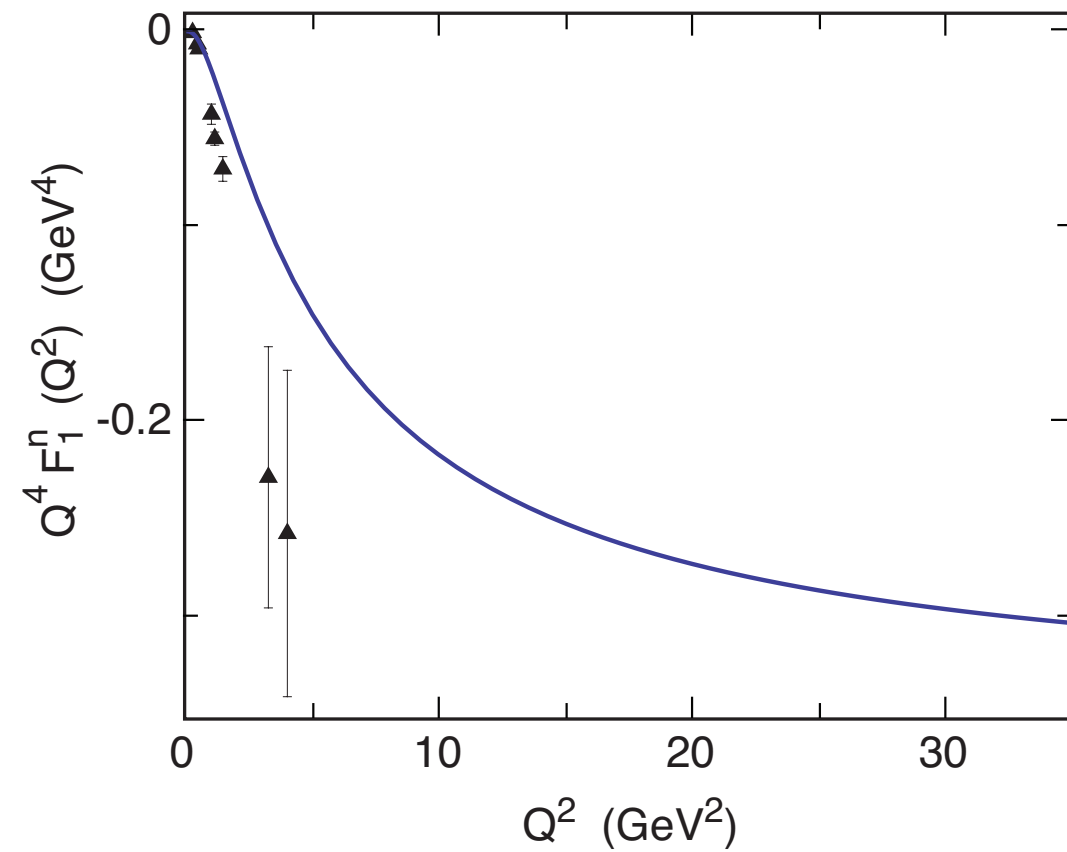
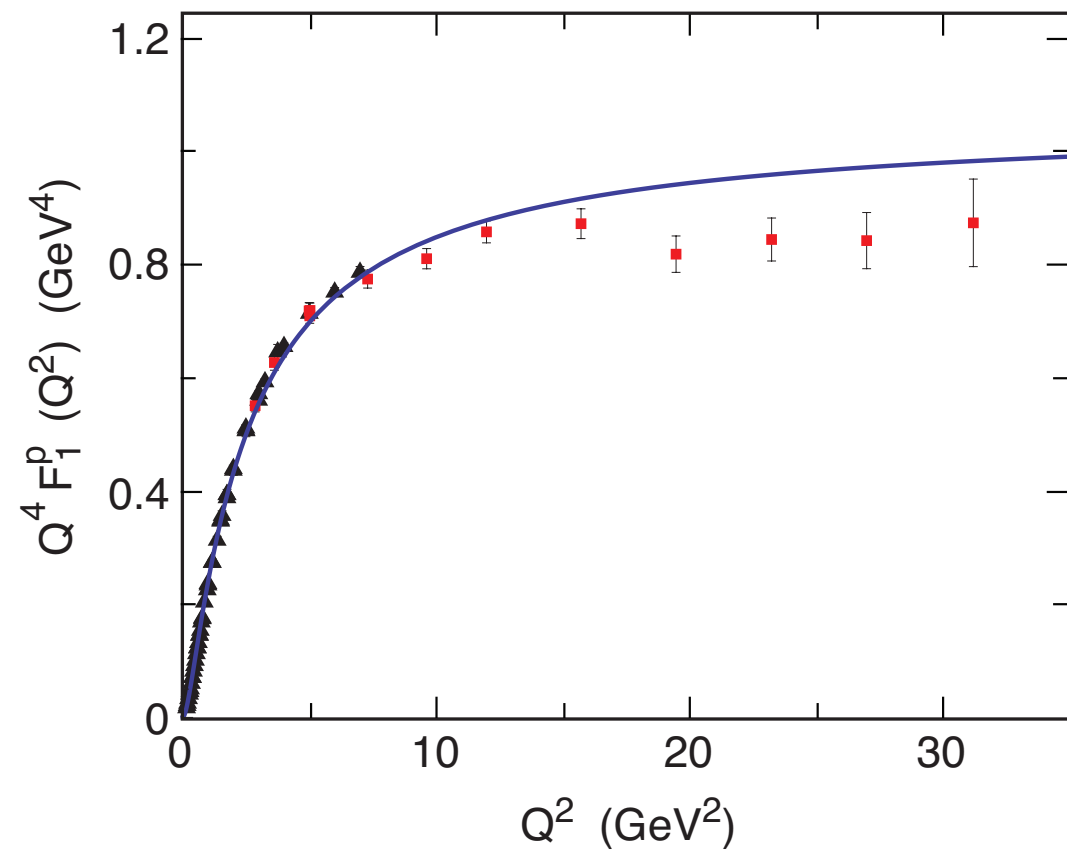
**14% four-quark
probability**

G. de Teramond & sjb

Pion Form Factor from AdS/QCD and Light-Front Holography



Using $SU(6)$ flavor symmetry and normalization to static quantities



Exact LF Formula for Pauli Form Factor

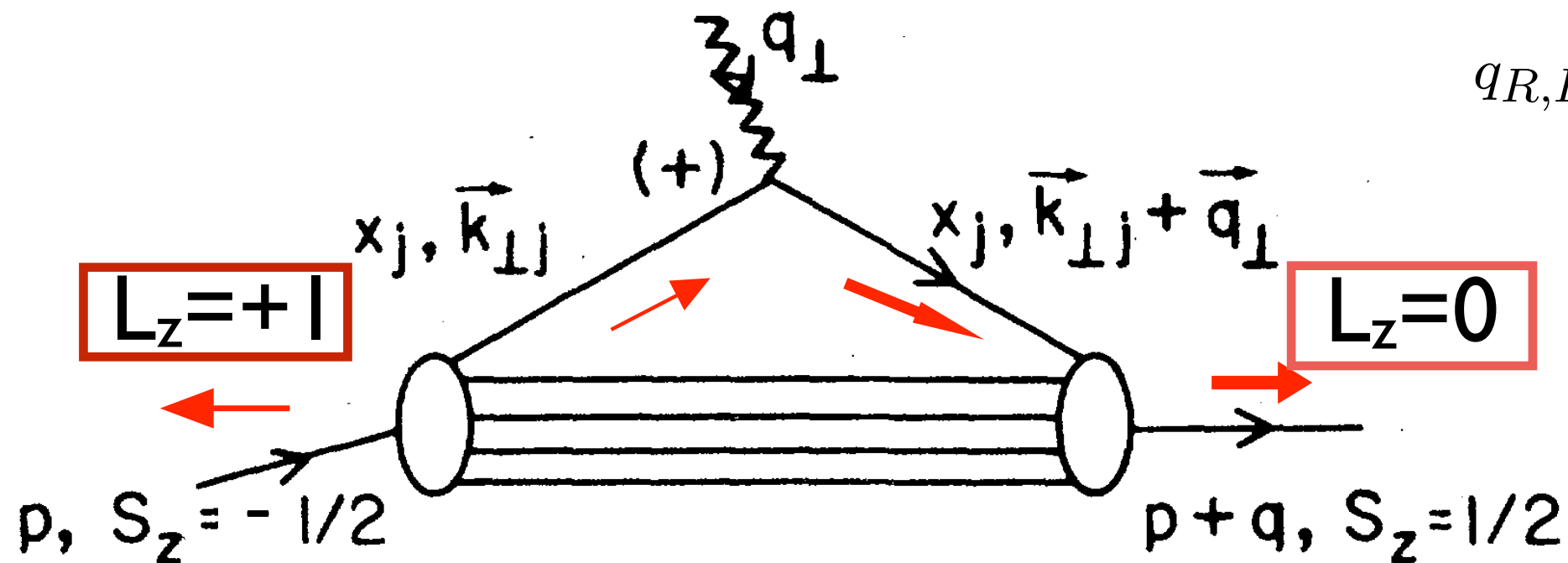
$$\frac{F_2(q^2)}{2M} = \sum_a \int [dx] [d^2\mathbf{k}_\perp] \sum_j e_j \frac{1}{2} \times \quad \text{Drell, sjb}$$

$$\left[-\frac{1}{q^L} \psi_a^{\uparrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R} \psi_a^{\downarrow*}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp$$

$$\mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j) \mathbf{q}_\perp$$

$$q_{R,L} = q^x \pm i q^y$$



Must have $\Delta \ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum

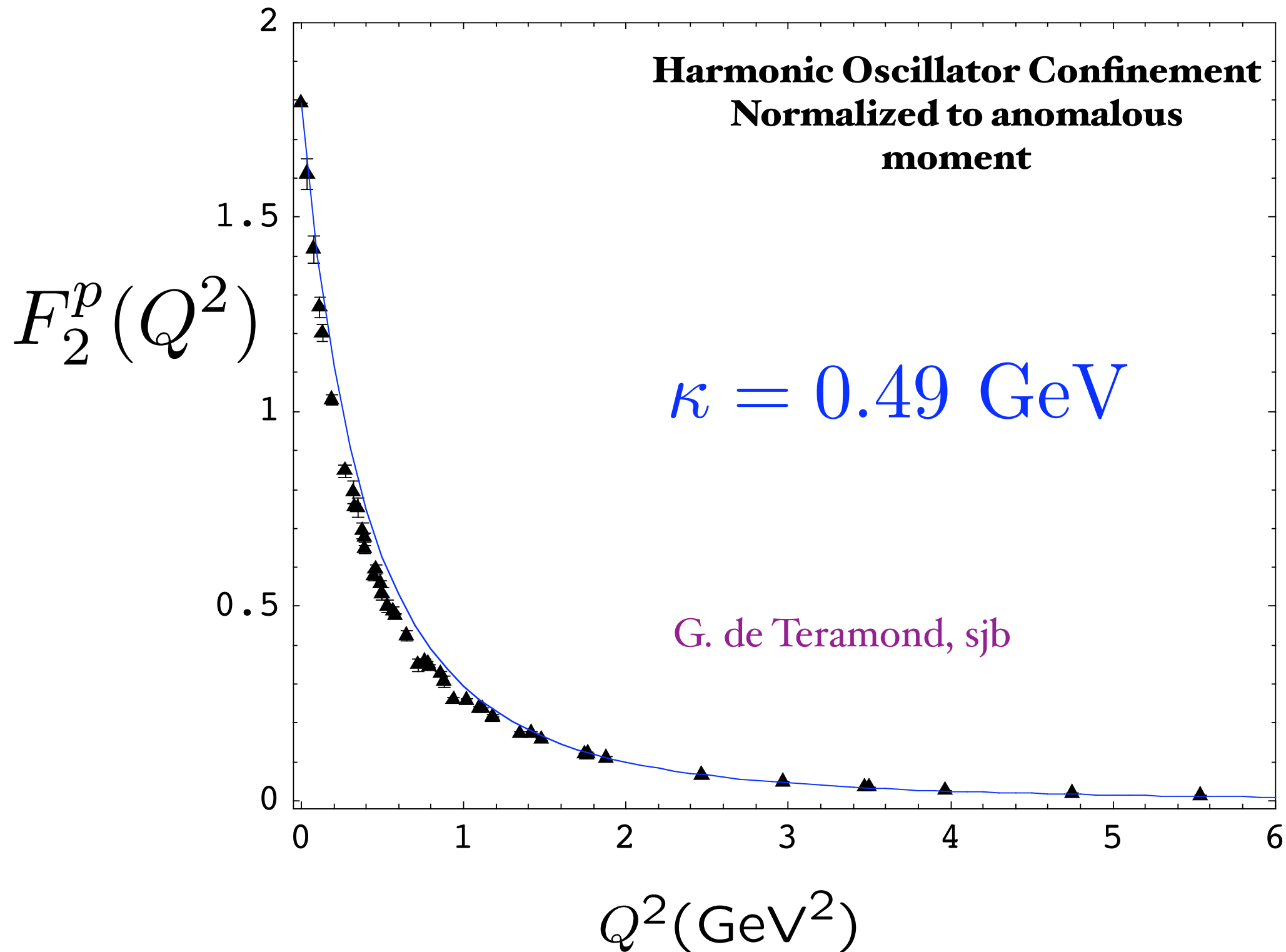
Spacelike Pauli Form Factor

From overlap of $L = 1$ and $L = 0$ LFWFs

Harmonic Oscillator Confinement
Normalized to anomalous
moment

$$\kappa = 0.49 \text{ GeV}$$

G. de Teramond, sjb

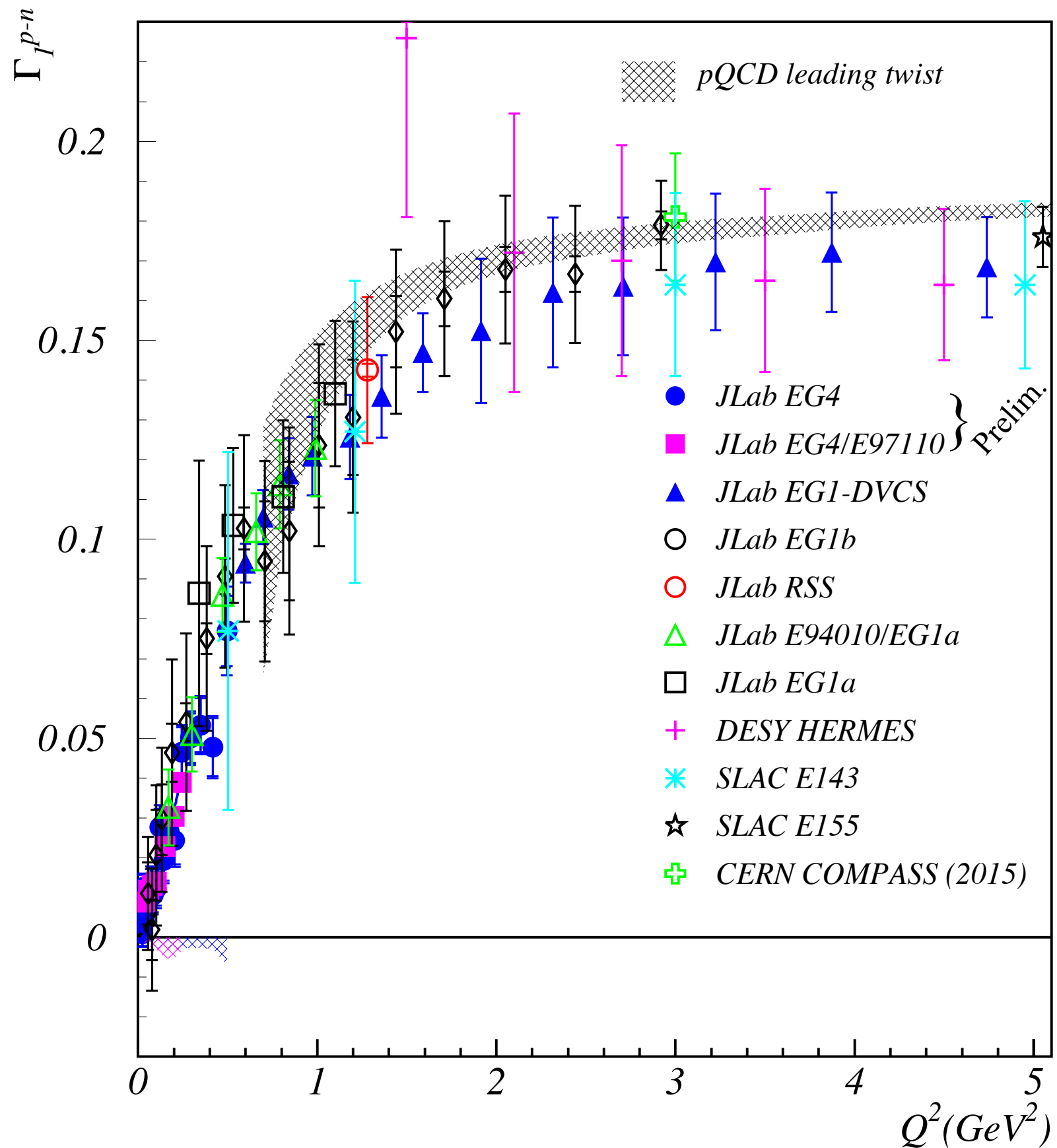


Bjorken sum rule defines effective charge: $\alpha_{g1}(Q^2)$

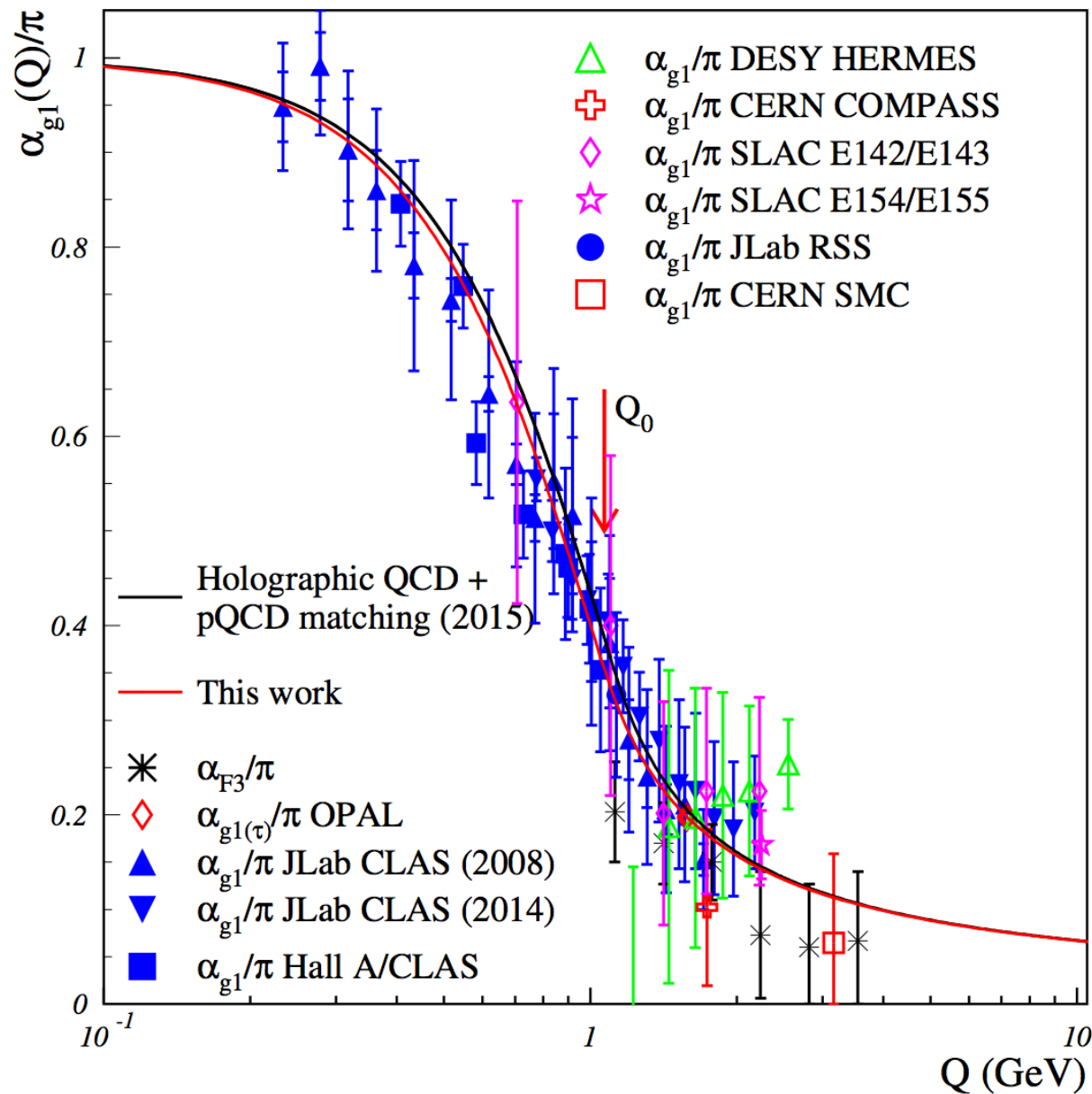
$$\int_0^1 dx [g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2)] \equiv \frac{g_a}{6} \left[1 - \frac{\alpha_{g1}(Q^2)}{\pi} \right]$$

- **Can be used as standard QCD coupling**
- **Well measured**
- **Asymptotic freedom at large Q^2**
- **Computable at large Q^2 in any pQCD scheme**
- **Universal β_0, β_1**
- **Analytic connection to other schemes:**
Commensurate scale relations

Bjorken sum Γ_1^{p-n} measurements



Running Coupling from AdS/QCD



Bjorken sum rule:

$$\frac{\alpha_{g_1}(Q^2)}{\pi} = 1 - \frac{6}{g_A} \int_0^1 dx g_1^{p-n}(x, Q^2)$$

Effective coupling in LFHQCD
(valid at low- Q^2)

$$\alpha_{g_1}^{AdS}(Q^2) = \pi \exp(-Q^2/4\kappa^2)$$

Imposing continuity for α
and its first derivative

A. Deur, S.J. Brodsky, G.F. de Téramond,
Phys. Lett. B 750, 528 (2015); J. Phys. G 44, 105005 (2017).

Analytic, defined at all scales, IR Fixed Point

$$m_\rho = \sqrt{2}\kappa$$

$$m_p = 2\kappa$$

Deur, de Tèramond, sjb

All-Scale QCD Coupling

Fit to Bj + DHG Sum Rules:
 $\kappa = 0.513 \pm 0.007 \text{ GeV}$

$$\frac{\alpha_{g_1}^s(Q^2)}{\pi}$$

$$e^{-\frac{Q^2}{4\kappa^2}}$$

Nonperturbative QCD
(Quark Confinement)

5-Loop β Prediction:
 $\Lambda_{\overline{MS}} = 0.339 \pm 0.019 \text{ GeV}$

Experiment:
 $\Lambda_{\overline{MS}} = 0.332 \pm 0.017 \text{ GeV}$

Use Q_0 for starting
DGLAP and ERBL
Evolution

Perturbative QCD
(Asymptotic Freedom)

Transition scale Q_0

$$Q_0 = 0.87 \pm 0.08 \text{ GeV}$$

$$\lambda \equiv \kappa^2$$

10^{-1}

1

10

Q (GeV)

Reverse Dimensional Transmutation!

Initial DGLAP evolution scale form IR-UV
matching of QCD coupling

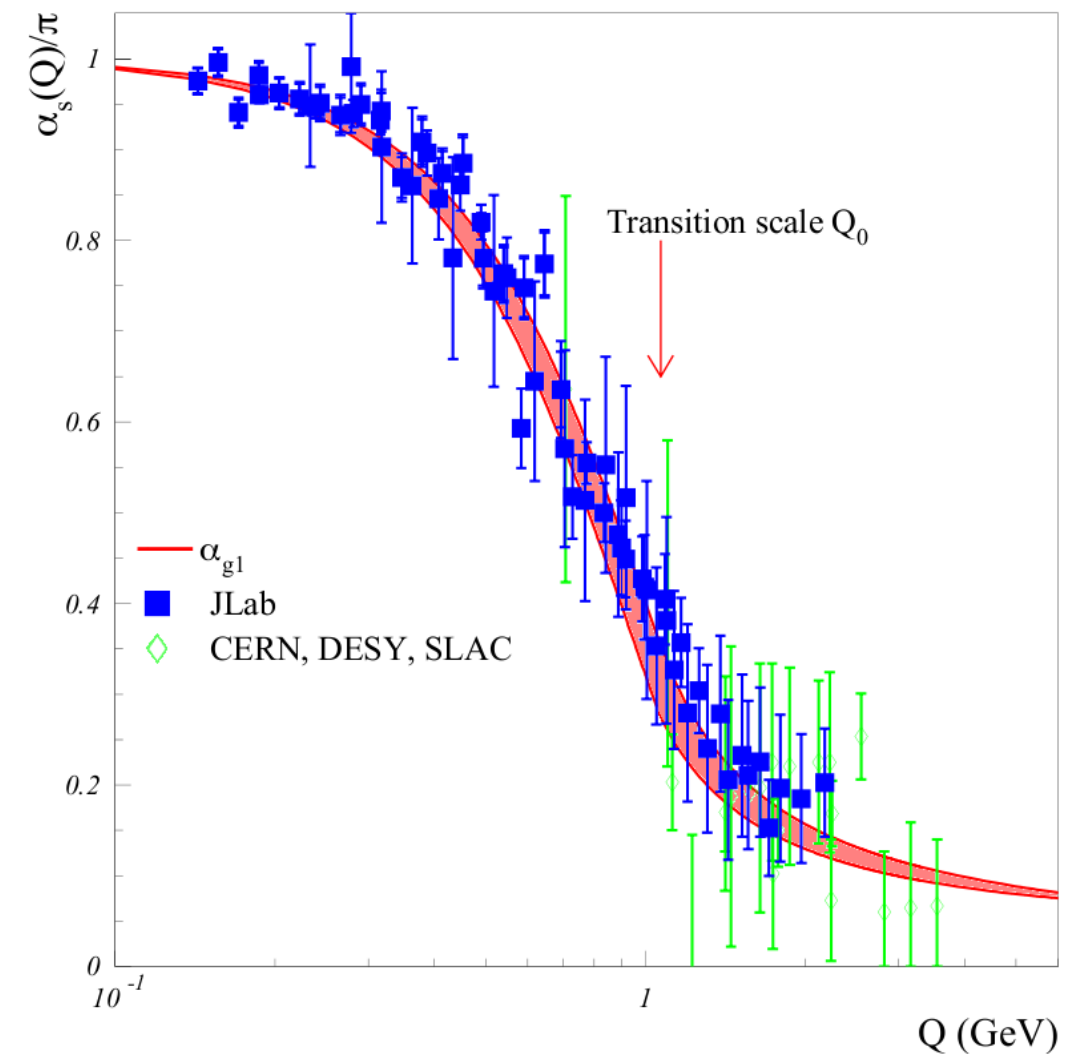
IR behavior of strong coupling in LFHQCD

$$\alpha_s^{IR}(Q^2) = \alpha_s^{IR}(0)e^{-Q^2/4\lambda}$$

Λ_{QCD} and transition scale Q_0 from matching
perturbative (5-loop) and nonperturbative
regimes for $\sqrt{\lambda} = 0.534 \pm 0.05$ GeV

Transition scale: $Q_0^2 \simeq 1 \text{ GeV}^2$

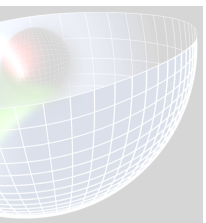
Connection between proton mass, $M_p^2 = 4\lambda$,
the ρ mass, $M_\rho^2 = 2\lambda$, and the perturbative
QCD scale Λ_{QCD} in any RS !

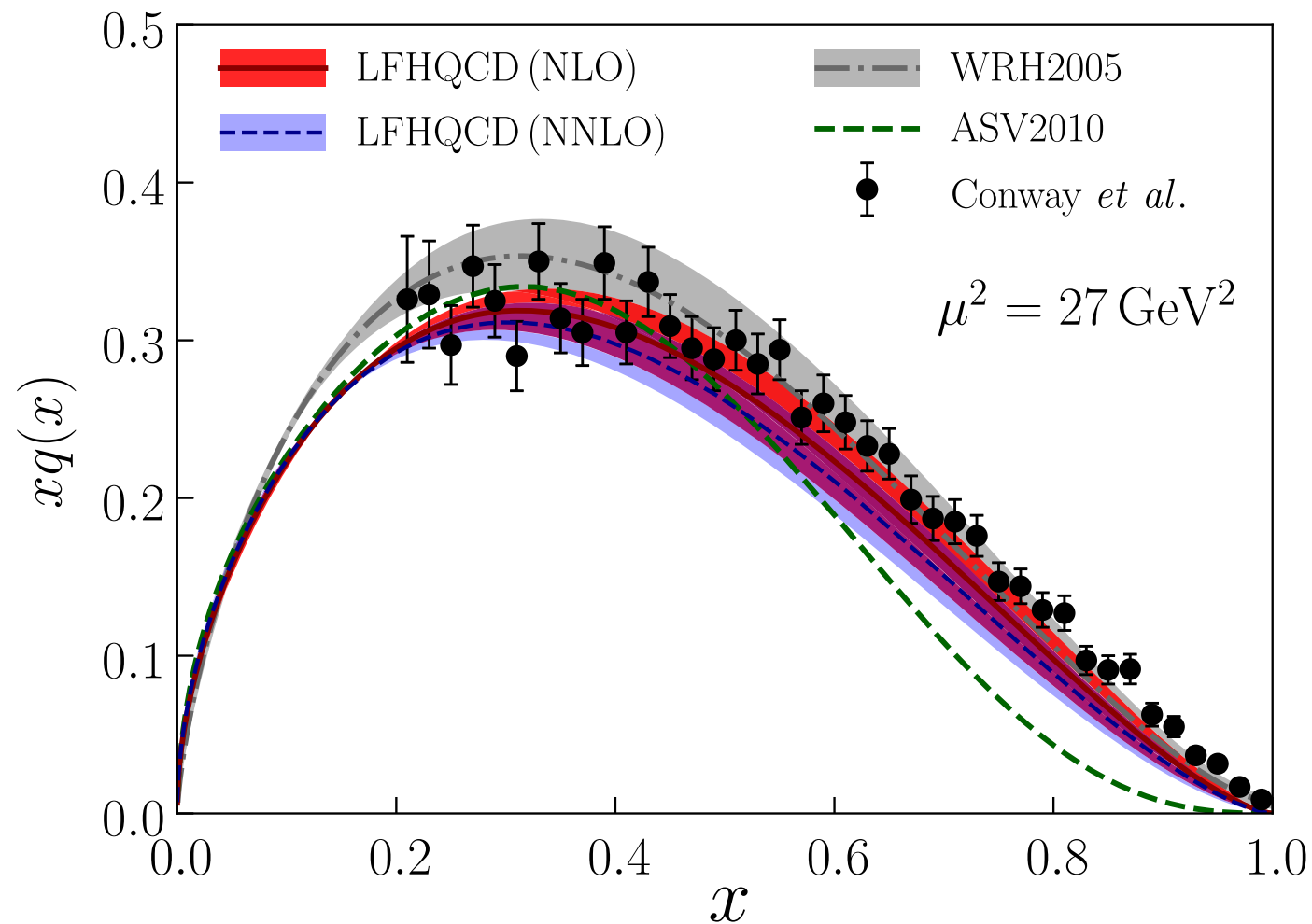


IR QCD strong coupling from Bjorken
sum-rule vs HLFQCD prediction (red)

Similar behavior of the IR coupling was obtained from the DSE

D. Binosi *et al.* (2017) and Z. F. Cui, *et al.* Chin. Phys. C **44**, 083102 (2020)

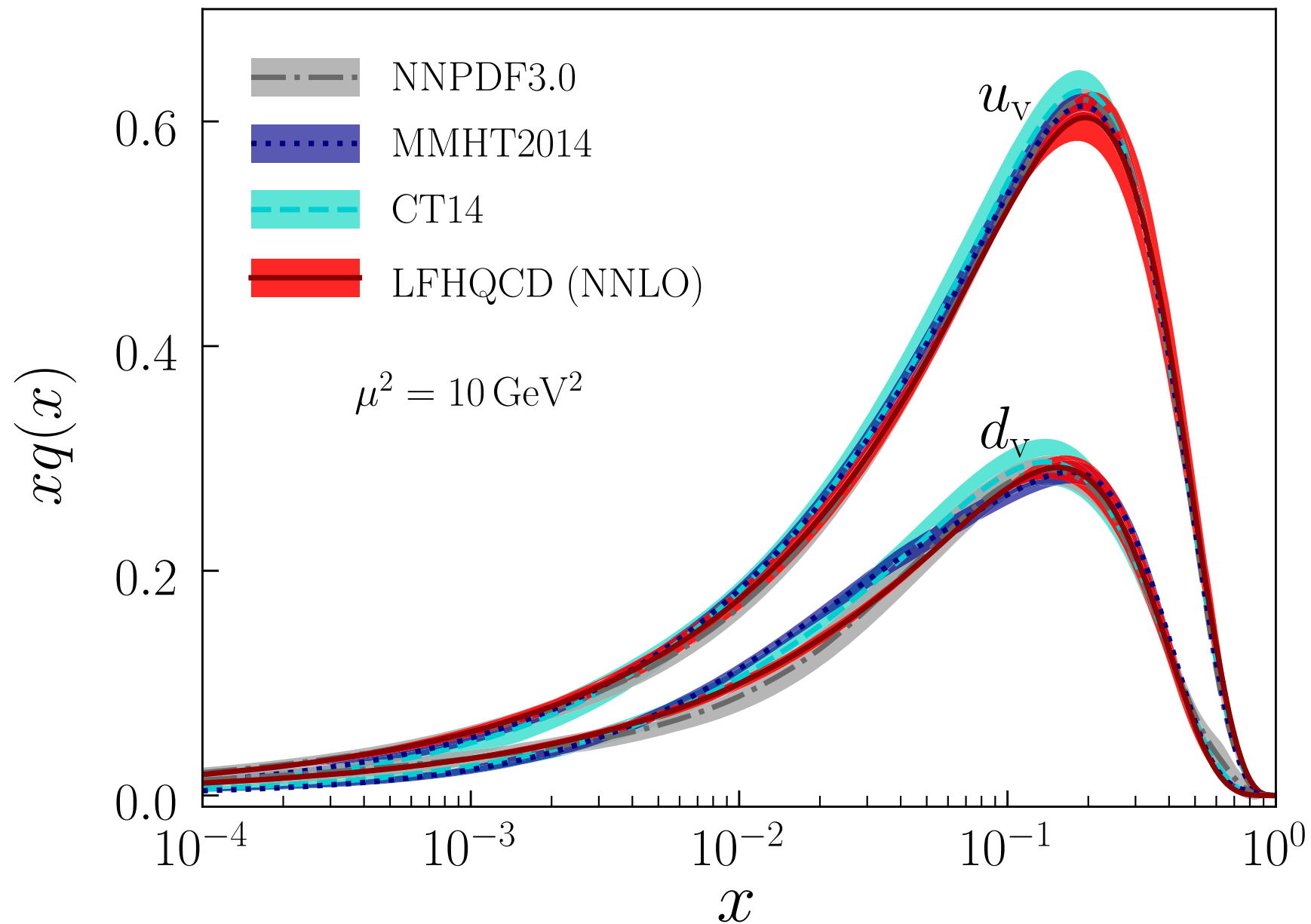




Comparison for $xq(x)$ in the pion from LFHQCD (red band) with the NLO fits [82,83] (gray band and green curve) and the LO extraction [84]. NNLO results are also included (light blue band). LFHQCD results are evolved from the initial scale $\mu_0 = 1.1 \pm 0.2 \text{ GeV}$ at NLO and the initial scale $\mu_0 = 1.06 \pm 0.15 \text{ GeV}$ at NNLO.

Universality of Generalized Parton Distributions in Light-Front Holographic QCD

Guy F. de Téramond, Tianbo Liu, Raza Sabbir Sufian, Hans Günter Dosch, Stanley J. Brodsky, and Alexandre Deur *PHYSICAL REVIEW LETTERS* 120, 182001 (2018)



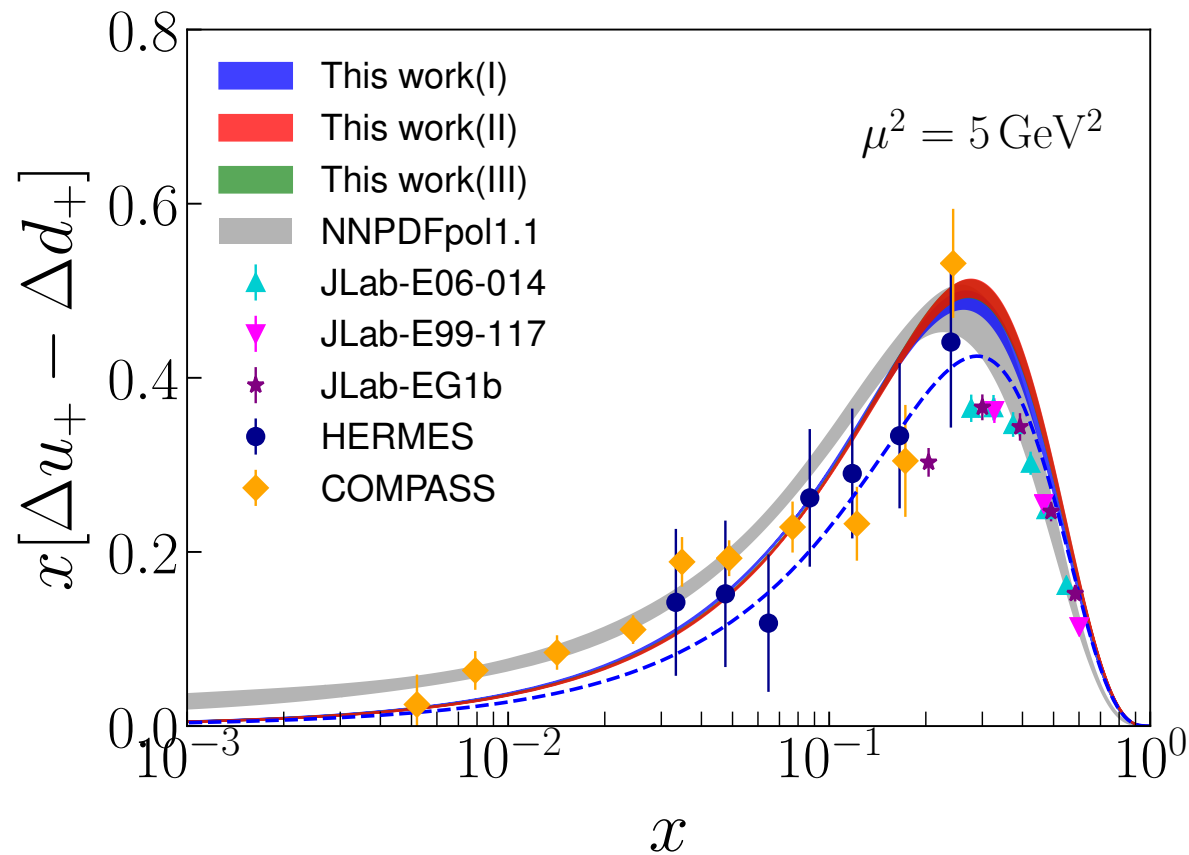
Comparison for $xq(x)$ in the proton from LFHQCD (red bands) and global fits: MMHT2014 (blue bands) [5], CT14 [6] (cyan bands), and NNPDF3.0 (gray bands) [77]. LFHQCD results are evolved from the initial scale $\mu_0 = 1.06 \pm 0.15$ GeV.

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PHYSICAL REVIEW LETTERS 120, 182001 (2018)

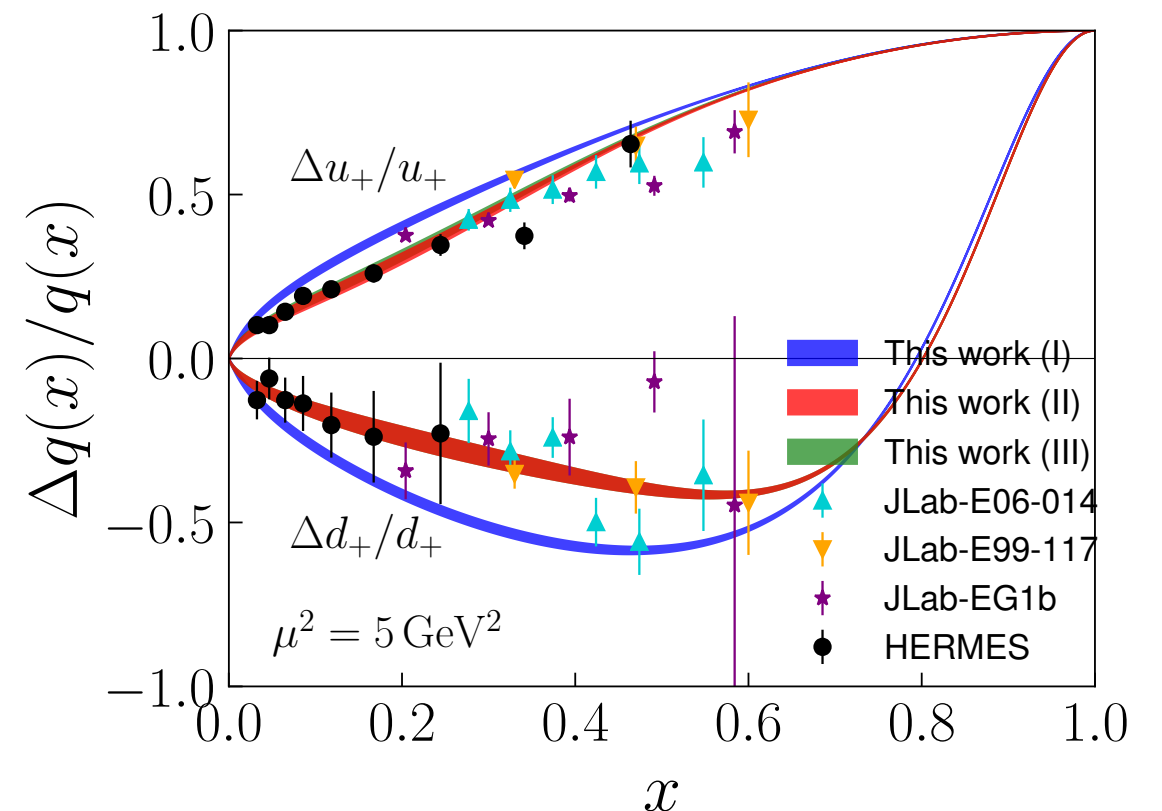
Tianbo Liu, Raza Sabbir Sufian, Guy F. de Te'ramond, Hans Gunter Dösch, Alexandre Deur, sjb



Polarized distributions for the
isovector combination $x[\Delta u_+(x) - \Delta d_+(x)]$

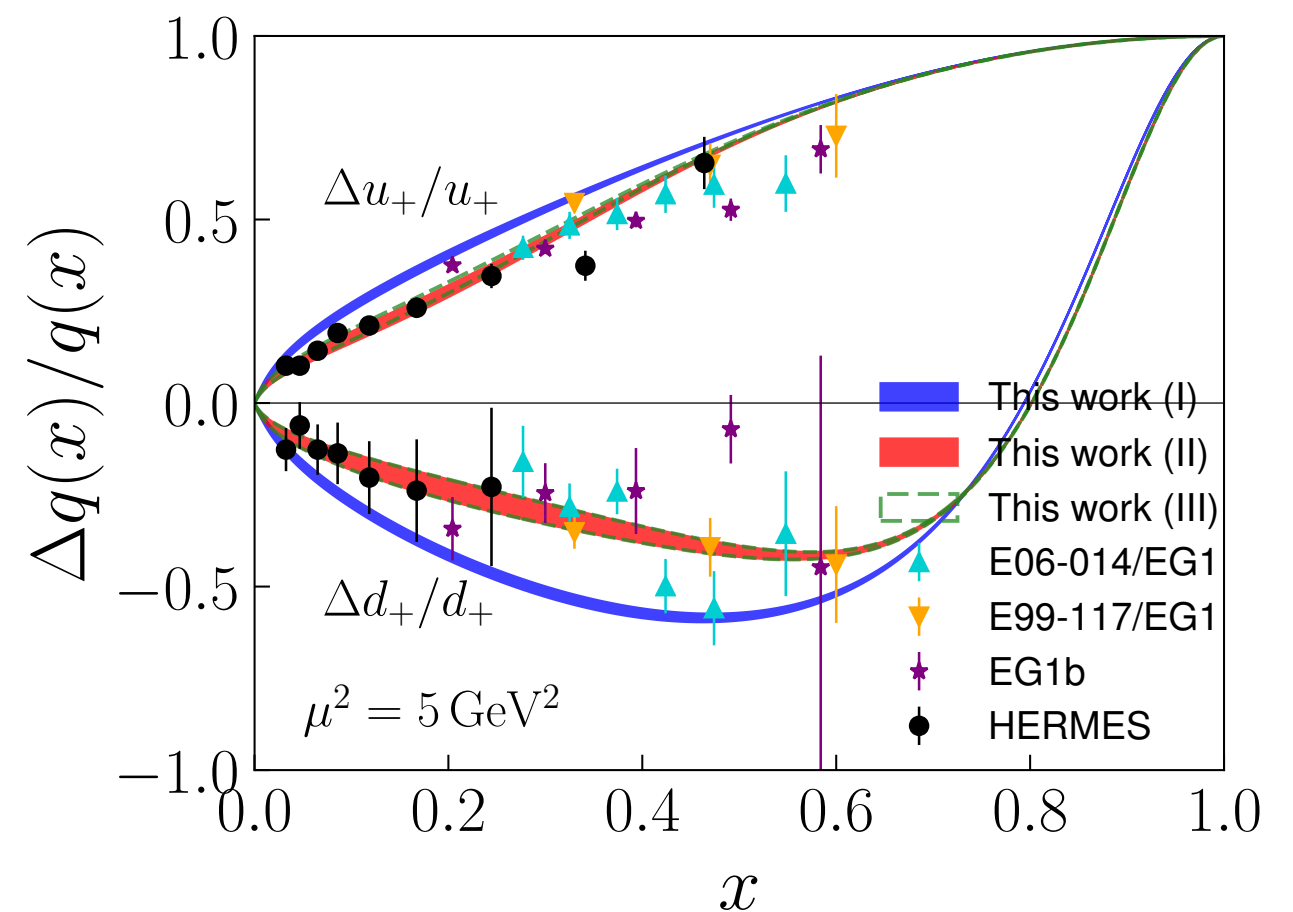
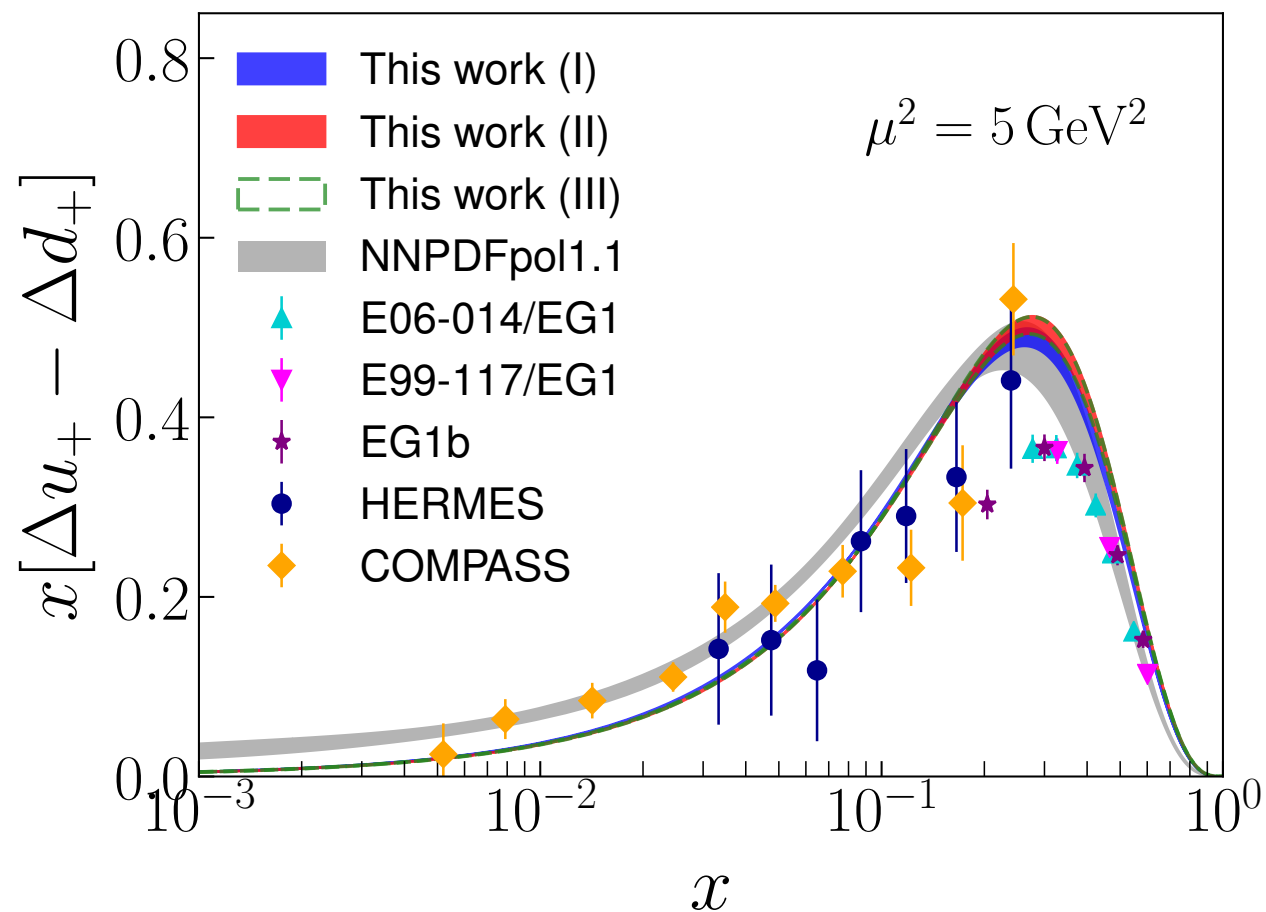
$$d_+(x) = d(x) + \bar{d}(x) \quad u_+(x) = u(x) + \bar{u}(x)$$

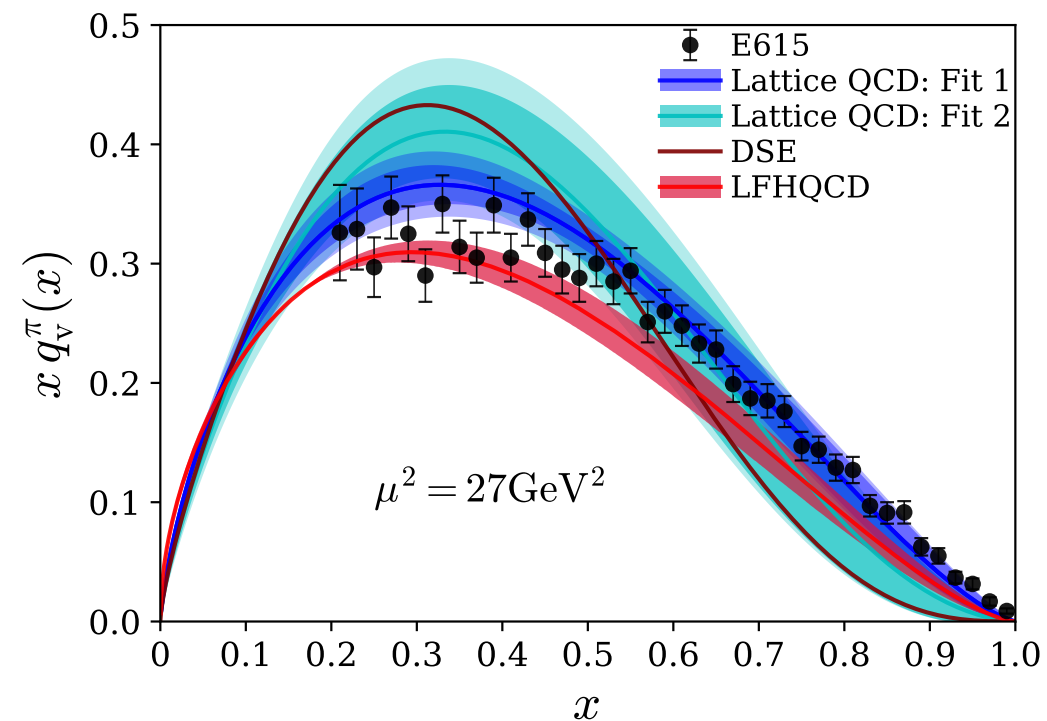
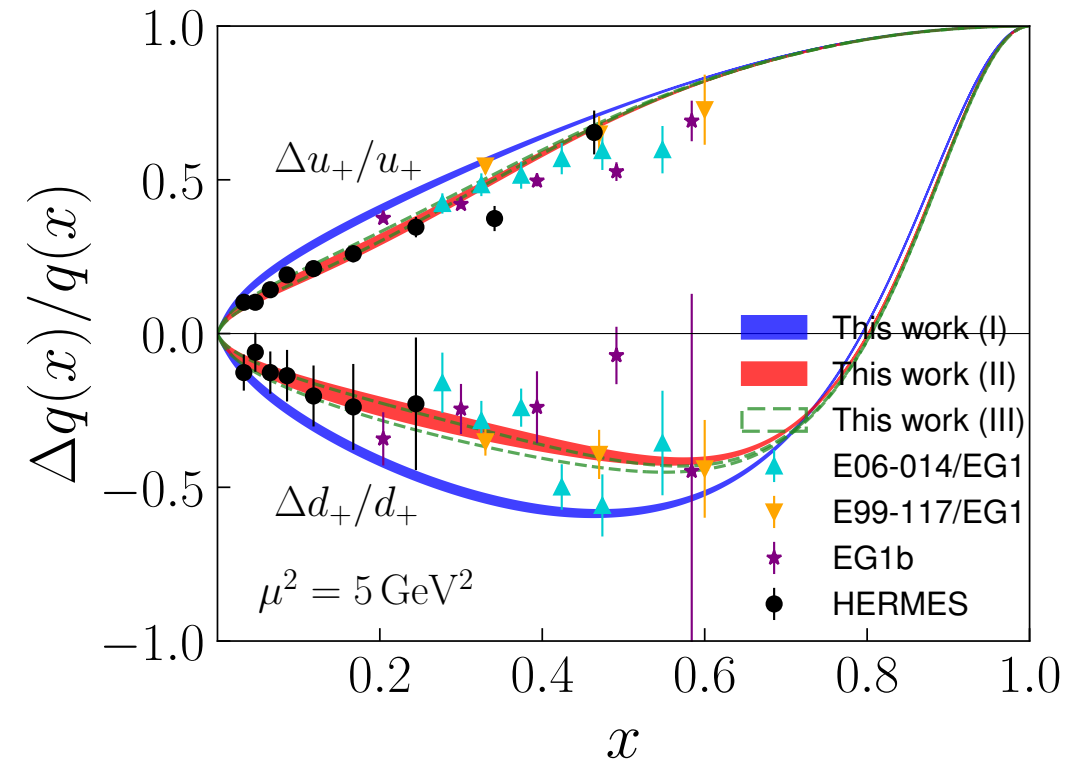
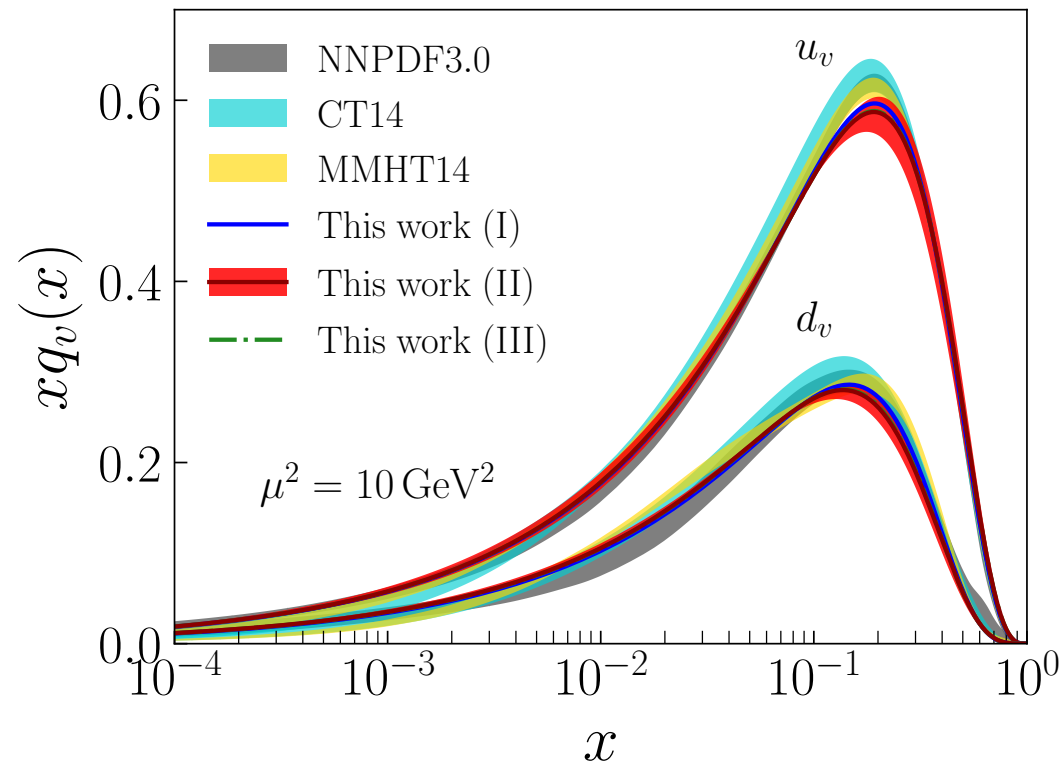
$$\Delta q(x) = q_{\uparrow}(x) - q_{\downarrow}(x)$$



Polarized GPDs and PDFs (HLFHS Collaboration, 2019)

- Separation of chiralities in the AdS action allows computation of the matrix elements of the axial current including the correct normalization, once the coefficients c_τ are fixed for the vector current
- Helicity retention between quark and parent hadron (pQCD prediction): $\lim_{x \rightarrow 1} \frac{\Delta q(x)}{q(x)} = 1$
- No spin correlation with parent hadron: $\lim_{x \rightarrow 0} \frac{\Delta q(x)}{q(x)} = 0$





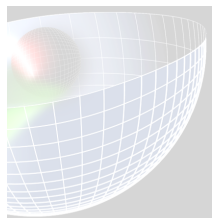
Separation of chiralities from the axial current
Coefficients c_τ are fixed from the vector current

Regge trajectory from HLFQCD

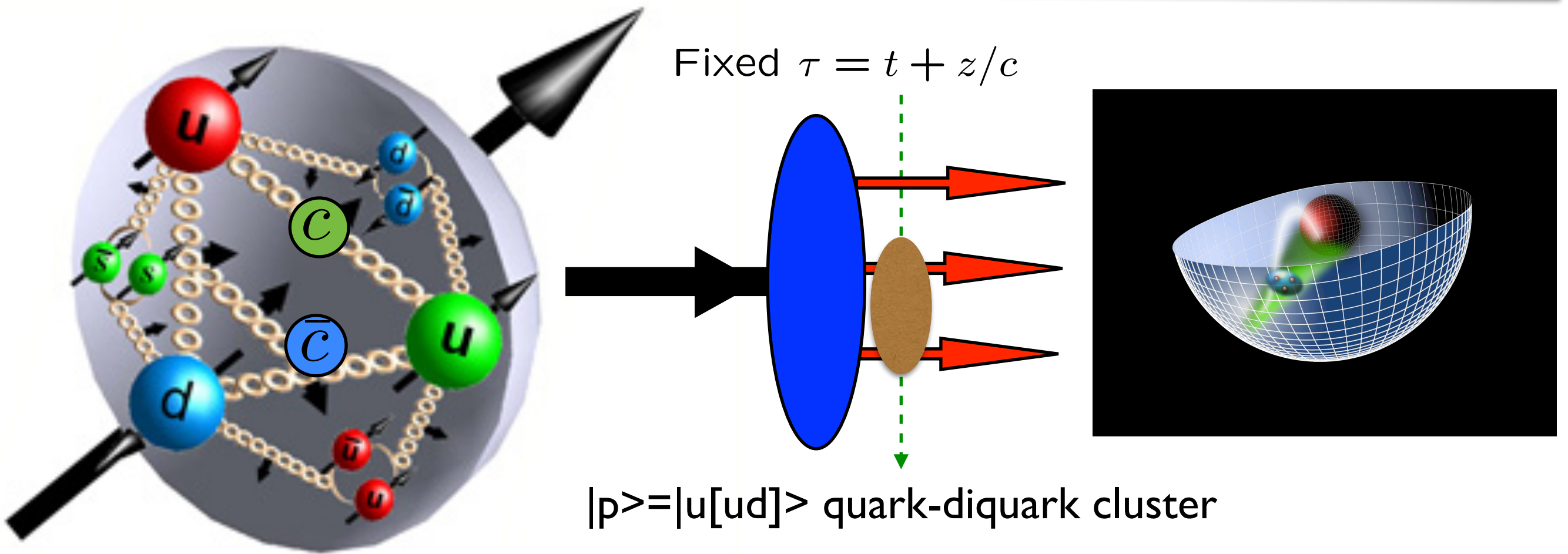
$$\alpha_A(t) = \frac{t}{4\lambda}$$

$$\lim_{x \rightarrow 1} \frac{\Delta q(x)}{q(x)} = 1, \quad \lim_{x \rightarrow 0} \frac{\Delta q(x)}{q(x)} = 0$$

DGLAP NNLO evolution from initial scale $\mu \simeq 1 \text{ GeV}$ from soft-hard matching in α_s



New Perspectives for Hadron Spectroscopy and Dynamics and the Running QCD Coupling from Color-Confining Holographic Light-Front QCD



with Guy de Tèramond, Hans Günter Dosch, Cédric Lorcè, Alexandre Deur, and Joshua Erlich

A.N. Mitra Memorial Symposium
April 15, 2025

Stan Brodsky

SLAC

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